MISSOULA COUNTY PUBLIC SCHOOLS

Forward Thinking, High Achieving.

PREK-12 MATHEMATICS CURRICULUM

Adopted June, 2013

This document was developed by the Missoula County Public Schools Curriculum Consortium, which includes Missoula County Public Schools, Hellgate Elementary School, and Target Range Elementary School Districts

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2013-14 MATH CONTENT AREA STANDING COMMITTEE

Name	Grade	School
Anderson, Crista	Instructional Coach	MCPS District
Beatty, Michelle	9-12	Willard Alternative High School
Bergeron, Michelle	K-8	Target Range District
Bolton, Tim	8	Meadow Hill Middle School
Brown, Lee	9-12	Hellgate High School
Brown, Rae	4	Hawthorne Elementary School
Bybee, Sarah	1	Hellgate Elementary District
Demmons, Seena	Instructional Coach	MCPS District
Erickson, David	Dept of C&I	University of Montana
Fisher Keffeler, Stacey	6	Washington Middle School
Giuliani, Denise	K	Rattlesnake Elementary School
Grant, Lori	Instructional Coach	MCPS District
Hanks, Heidi	6	Hellgate Elementary School
Harris, Sherrie	2	Paxson Elementary School
Holden, Diane	2	Cold Springs Elementary School
Hopper, Megan	K-8	Target Range Elementary District
Joyner-O'Connor, Kimberly	3	Lewis and Clark Elementary School
Kattell, Nita	5	Hellgate Elementary District
Kral Kemmis, Erin	K	Lewis and Clark Elementary School
Laslovich, Kacie	6	Meadow Hill Middle School
Laslovich, Luke	Principal	Target Range Elementary District
Lindburg, Shirley	Gifted Ed Coordinator	MCPS District
Little, Gary	9-12	Sentinel High School
Lokey, Alison	K-8	Target Range Elementary District
Lynn, Melissa	1	Lowell Elementary District
Matosich, Craig	9-12	Sentinel High School
McHugh, Casey	5	Cold Springs Elementary School
McHugh, Tim	9-12	Big Sky High School
Meyers, Kammy	1	Franklin Elementary School
Mooney, DeeAnn	9-12	Big Sky High School
Mueller, Susie	5	Russell Elementary School
Murphy, Cassandra	K	Chief Charlo Elementary School
Nisbet, Jamie	1	Hawthorne Elementary School
Notti, Melissa	3	Lowell Elementary School
Nuttall, Kye	8	C.S. Porter Middle School
Nuttall, Robyn	Instructional Coach	MCPS District
Pernell, Stephanie	4	Rattlesnake Elementary School
Raser, Holly	K-8	Target Range District
Reschke, Tammy	5	Paxson Elementary School
Schowengerdt, Angela	7	Washington Middle School
Sharkey, Betsy	1/2	Lewis and Clark Elementary School
Shepherd, Nicholas	7	C.S. Porter Middle School

2013-14 MATH CONTENT AREA STANDING COMMITTEE

Name	Grade	School
Simianer, Melissa	K-8	Target Range Elementary District
Startin, Alexandra	9-12	Hellgate High School
Stone, Mary	9-12	Seeley-Swan High School
Sullivan, Shaleen	1	Chief Charlo Elementary School
Tackett, Diana	4	Paxson Elementary School
Toller, Teresa	8	Washington Middle School
Vaneps, Alanna	Curriculum Coordinator	MCPS District
Wilson, Sharon	K-5 Resource	Russell Elementary School

MISSION

At Missoula County Public Schools (MCPS), our mission is to ensure that each student achieves his/her full and unique potential. (*Approved by Board of Trustees 2009*)

VISION

MCPS provides a broad education, recognized for its quality, for every student in a safe, stimulating learning environment. All MCPS students are challenged to develop critical thinking skills, citizenship responsibilities, communication competency, value for the arts, literature, and sciences, understanding of the importance of health and wellness, a love for learning, and preparation for life beyond high school regardless of their vocational pathway. The community trusts and supports the MCPS Board of Trustee's leadership and vision because the Board: Seeks out and values input from the community through useful public participation strategies and is known for fiscal responsibility and efficiency. Hires highly qualified and competent administration and staff and encourages ongoing education for them as well as Board members. Searches out and is successful at finding alternative and non-traditional funding sources to support District programs. Is perceived by the public as competent, consistent and having integrity.

MATHEMATICS CURRICULUM PHILOSOPHY

Students today require mathematical skills that surpass what was needed in the past. The Montana Common Core Mathematics Standards, adopted by MCPS, establish a national benchmark of skills required to compete globally in the 21st Century. These Standards reflect accepted research-based learning progressions detailing what is known about how students' mathematical knowledge, skill, and understanding develop over time. The rigor of the Montana Common Core Standards build deeper understanding at each grade level to equip students to interpret information, apply the mathematics, and communicate a justifiable outcome when solving problems.

The MCPS Mathematics Curriculum focuses on the mathematical concepts developed by students at all levels. The standards are consistent with the MCPS 21st Century Model of Education, the NCTM Standards, and the Montana Common Core State Standards for Mathematics.

HIGH QUALITY MATHEMATICS TEACHING & LEARNING

Our district's vision is that mathematical learning should be explored in ways that stimulate active engagement, create enjoyment of mathematics, and develop depth of understanding to promote 21st Century learning.

All students will understand and apply the eight mathematical practices embedded in the Montana Common Core Standards deeply and effectively. The standards allow students to grow in mathematical maturity, expertise, and understanding through the elementary, middle, and high school years. To achieve mathematical understanding, students should have a balance of mathematical procedures and conceptual understanding. Students should be actively engaged in doing meaningful mathematical tasks, math discussion, and applying mathematics in interesting, thought-provoking situations. Learning should be designed to challenge students in multiple ways. Activities should enable students to solve progressively deeper, broader, and more sophisticated problems which increase mathematical reasoning.

GUIDING PRINCIPLES

The following principles are philosophical statements that underlie the mathematics content and practice standards, and resources. They should guide the construction and evaluation of mathematics instruction.

These recommended Guiding Principles for Mathematics Programs are adapted from the Massachusetts Curriculum Framework January 2011; http://www.doe.mass.edu/frameworks/current.html

Guiding Principle 1: LEARNING	**Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.**

Students need to understand mathematics deeply and use it effectively. The standards of mathematical practice describe ways in which students increasingly engage with the subject matter as they grow in mathematical maturity and expertise through the elementary, middle, and high school years.

To achieve mathematical understanding, students should have a balance of mathematical procedures and conceptual understanding. Students should be actively engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought-provoking situations. Student understanding is further developed through ongoing reflection about cognitively demanding and worthwhile tasks.

Tasks should be designed to challenge students in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program. Activities should build upon curiosity and prior knowledge, and enable students to solve progressively deeper, broader, and more sophisticated problems. Mathematical tasks reflecting sound and significant mathematics should generate active classroom talk, promote the development of conjectures, and lead to an understanding of the necessity for mathematical reasoning.

Guiding Principle 2: TEACHING	An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

The sequence of topics and performances should be based on what is known about how students' mathematical knowledge, skill, and understanding develop over time. What and how students are taught should reflect not only the topics within mathematics but also the key ideas that determine how knowledge is organized and generated within mathematics. Students should be asked to apply their learning and to show their mathematical thinking and understanding by engaging in the first Mathematical Practice, Making sense of problems and persevere in solving them. This requires teachers who have a deep knowledge of mathematics as a discipline.

Mathematical problem solving is the hallmark of an effective mathematics program. Skill in mathematical problem solving requires practice with a variety of mathematical problems as well as a firm grasp of mathematical techniques and their underlying principles. Armed with this deeper knowledge, the student can then use mathematics in a flexible way to attack various problems and devise different ways of solving any particular problem. Mathematical problem solving calls for reflective thinking, persistence, learning from the ideas of others, and going back over one's own work with a critical eye. Students should construct viable arguments and critique the reasoning of others, they analyze situations and justify their conclusions and are able to communicate them to others and respond to the arguments of others. (See Mathematical Practice 3, Construct viable arguments and critique reasoning of others.) Students at all grades can listen or read the arguments of others and decide whether they make sense, and ask questions to clarify or improve the arguments.

Mathematical problem solving provides students with experiences to develop other mathematical practices. Success in solving mathematical problems helps to create an abiding interest in mathematics. Students learn to model with mathematics, they learn to apply the mathematics that they know to solve problems arising in everyday life, society, or the workplace. (See Mathematical Practice 4, Model with mathematics.)

For a mathematics program to be effective, it must also be taught by knowledgeable teachers. According to Liping Ma, "The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics."1A landmark study in 1996 found that students with initially comparable academic achievement levels had vastly different academic outcomes when teachers' knowledge of the subject matter differed.2 The message from the research is clear: having knowledgeable teachers really does matter; teacher expertise in a subject drives student achievement. "Improving teachers' content subject matter knowledge and improving students' mathematics education are thus interwoven and interdependent processes that must occur simultaneously."3

Guiding Principle 3: TECHNOLOGY	Technology is an essential tool that should be used strategically in mathematics education.

Technology enhances the mathematics curriculum in many ways. Tools such as measuring instruments, manipulatives (such as base ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for developing and applying mathematical concepts. However, appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills. Elementary students should learn how to perform the basic arithmetic operations independent of the use of a calculator.4 Although the use of a graphing calculator can help middle and secondary students to visualize properties of functions and their graphs, graphing calculators should be used to enhance their understanding and skills rather than replace them.

Teachers and students should consider the available tools when presenting or solving a problem. Student should be familiar with tools appropriate for their grade level to be able to make sound decisions about which of these tools would be helpful. (See Mathematical Practice 5, Use appropriate tools strategically.)

Technology enables students to communicate ideas within the classroom or to search for information in external databases such as the Internet, an important supplement to a school's internal library resources. Technology can be especially helpful in assisting students with special needs in regular and special classrooms, at home, and in the community.

Technology changes what mathematics is to be learned and when and how it is learned. For example, currently available technology provides a dynamic approach to such mathematical concepts as functions, rates of change, geometry, and averages that was not possible in the past. Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it.

Guiding Principle 4: EQUITY	All students should have a high quality mathematics program that prepares them for college and a career.

All students should have high quality mathematics programs that meet the goals and expectations of these standards and address students' individual interests and talents. The standards provide clear signposts along the way to the goal of college and career readiness for all students. The standards provide for a broad range of students, from those requiring tutorial support to those with talent in mathematics. To promote achievement of these standards, teachers should encourage classroom talk, reflection, use of multiple problem solving strategies, and a positive disposition toward mathematics. They should have high expectations for all students. At every level of the education system, teachers should act on the belief that every child should learn challenging mathematics. Teachers and guidance personnel should advise students and parents about why it is important to take advanced courses in mathematics and how this will prepare students for success in college and the workplace.

All students should have the benefit of quality instructional materials, good libraries, and adequate technology. All students must have the opportunity to learn and meet the same high standards. In order to meet the needs of the greatest range of students, mathematics programs should provide the necessary intervention and support for those students who are below- or above grade-level expectations. Practice and enrichment should extend beyond the classroom. Tutorial sessions, mathematics clubs, competitions, and apprenticeships are examples of mathematics activities that promote learning.

Because mathematics is the cornerstone of many disciplines, a comprehensive curriculum should include applications to everyday life and modeling activities that demonstrate the connections among disciplines. Schools should also provide opportunities for communicating with experts in applied fields to enhance students' knowledge of these connections.

An important part of preparing students for college and careers is to ensure that they have the necessary mathematics and problem-solving skills to make sound financial decisions that they face in the world every day, including setting up a bank account; understanding student loans; credit and debit; selecting the best buy when shopping; choosing the most cost effective cell phone plan based on monthly usage; and so on.

Guiding Principle 5: LITERACY ACROSS THE CONTENT AREAS	**An effective mathematics program builds upon and develops students' literacy skills and knowledge.**

Reading, writing, and communication skills are necessary elements of learning and engaging in mathematics, as well as in other content areas. Supporting the development of students' literacy skills will allow them to deepen their understanding of mathematics concepts and help them determine the meaning of symbols, key terms, and mathematics phrases as well as develop reasoning skills that apply across the disciplines. In reading, teachers should consistently support students' ability to gain and deepen understanding of concepts from written material by acquiring comprehension skills and strategies, as well as specialized vocabulary and symbols. Mathematics classrooms should make use of a variety of text materials and formats, including textbooks, math journals, contextual math problems, and data presented in a variety of media.

In writing, teachers should consistently support students' ability to reason and deepen understanding of concepts and the ability to express them in a focused, precise, and convincing manner. Mathematics classrooms should incorporate a variety of written assignments ranging from math journals to formal written proofs. In speaking and listening, teachers should provide students with opportunities for mathematical discourse, to use precise language to convey ideas, to communicate a solution, and support an argument.

Guiding Principle 6: ASSESSMENT	Assessment of student learning in mathematics should take many forms to inform instruction and learning.

A comprehensive assessment program is an integral component of an instructional program. It provides students with frequent feedback on their performance, teachers with diagnostic tools for gauging students' depth of understanding of mathematical concepts and skills, parents with information about their children's performance in the context of program goals, and administrators with a means for measuring student achievement.

Assessments take a variety of forms, require varying amounts of time, and address different aspects of student learning. Having students "think aloud" or talk through their solutions to problems permits identification of gaps in knowledge and errors in reasoning. By observing students as they work, teachers can gain insight into students' abilities to apply appropriate mathematical concepts and skills, make conjectures, and draw conclusions. Homework, mathematics journals, portfolios, oral performances, and group projects offer additional means for capturing students' thinking, knowledge of mathematics, facility with the language of mathematics, and ability to communicate what they know to others. Tests and quizzes assess knowledge of mathematical facts, operations, concepts, and skills and their efficient application to problem solving. They can also pinpoint areas in need of more practice or teaching. Taken together, the results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

Assessment should also be a major component of the learning process. As students help identify goals for lessons or investigations, they gain greater awareness of what they need to learn and how they will demonstrate that learning. Engaging students in this kind of goal-setting can help them reflect on their own work, understand the standards to which they are held accountable, and take ownership of their learning.

NATIONAL COUNCIL FOR TEACHERS OF MATHEMATICS: GUIDING PRINCIPLES

- **Curriculum must be a collection of activities which are coherent, focused on important mathematics, and well-articulated across the grades.**
    - **Focus and coherence:** Mathematics consists of different topical strands, such as algebra and geometry, which are highly interconnected. The curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on or connect with other ideas, thus enabling students to learn with understanding, develop skill proficiency, and solve problems.
    - **Important mathematics:** A mathematics curriculum should focus on mathematics content and processes that are important and worth the time and attention of students. Mathematics topics are important for different reasons: developing other mathematical ideas, linking different areas of mathematics and in preparing students for college, the workforce, and citizenship.
    - **Articulation across grades:** Learning mathematics involves accumulating ideas and building successively deeper and more refined understanding. The curriculum must emphasize *depth* over breadth and must focus on the *essential ideas* and *processes of mathematics*. It must be focused in scope and delve deeply into each topic and concept, and is coherent across grades. Mathematical literacy emerges from, among other foundational understandings, a mature sense of number that includes an understanding of place value and comfort with estimating; a data sense that recognizes outliers and misinterpretation of data; a spatial sense that links two- and three-dimensional objects; and a symbol sense that results in algebraic representations that enable generalizations and predictions.

- **Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Learning mathematics with understanding is essential.**

- **The topics studied in the curriculum must be taught and learned in an equitable manner in a setting that ensures that problem solving, reasoning, connections, communication, and conceptual understanding are all developed simultaneously along with procedural fluency.**
    - **Problem Solving:** Problem solving means engaging in a task for which the solution method is not known in advance. To find a solution, students must draw on their knowledge. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort. They should be encouraged to reflect on their thinking.
    - **Reasoning and Proof:** Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove. By developing ideas, exploring phenomena, justifying results, and using

mathematical conjectures in all content areas and at *all grade levels*, students should recognize and expect that mathematics makes sense.

  - **Communication:** Communicating mathematical thinking and reasoning is an essential part of developing understanding, sharing, and clarifying ideas. When students are challenged to think and reason about mathematics and communicate the results of their thinking with others, they learn to be clear and convincing in their verbal and written explanations. Listening to others explain gives students opportunities to develop their own understanding. Conversations in which mathematical ideas are explored from multiple perspectives help learners sharpen their ability to reason, conjecture, and make connections.
  - **Connections:** Through everyday experiences, students should recognize and use connections among mathematical ideas. Of great importance are the infinite connections between algebra and geometry. These two strands of mathematics are mutually reinforcing in terms of concept development and the results that form the basis for much advanced work in mathematics as well as in applications. Students need experiences applying mathematics concepts and representations to describe and predict events in almost all academic disciplines, as well as in the workplace as we develop a fully informed citizenry.

- **Content should include the following key content areas.**
  - **Number and Operations with Procedural Fluency:** Proficiency with number and operations requires the deep and fundamental understanding of counting numbers, rational numbers (fractions, decimals, and percents), and positive and negative numbers, beginning in the elementary and middle grades. This understanding is extended to other number systems. Students must demonstrate understanding of numbers and relationships among numbers with a focus on the place-value system. Students must develop understanding of number operations and how they relate to one another.
    Written mathematical procedures—computational procedures in the elementary grades and more symbolic algebraic procedures as students move into the secondary level—continue to be an important focus of school mathematics programs. Students need to be comfortable and competent with estimation and mental math. They should become proficient at using mental math shortcuts, performing basic computations mentally, and generating reasonable estimates for situations involving size, distance, and magnitude.

  - **Algebra:** Algebra is more than a set of procedures for manipulating symbols. Algebra provides a way to explore, analyze, and represent mathematical concepts and ideas. It can describe relationships that mathematical or that arises in real-world phenomena and is modeled by algebraic expressions. Learning algebra helps students make connections in varied mathematical representations, mathematics topics, and disciplines that rely on mathematical relationships Algebraic concepts and skills should be a focus *across the pre-K–12 curriculum*. The development of algebraic concepts and skills does not occur within a single course or academic year. An understanding of algebra as a topic is a course of study. At the elementary school level, teachers help students be proficient with numbers, identify relationships, and use a variety of representations to describe and generalize patterns and solve equations. Secondary school teachers help students move from verbal descriptions of relationships to proficiency in the language of functions and skill in generalizing numerical relationships expressed by symbolic representations.

- **Geometry and Measurement:** Geometry is a natural place for the development of students' reasoning and justification skills, culminating in work with proof in the secondary grades. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and support in problem solving. Geometric representations help students make sense of area and fractions; histograms and scatter plots can give insights about data; and coordinate graphs can serve to connect ideas in geometry and algebra.
  The study of measurement is important in the mathematics curriculum from prekindergarten through high school because of the practicality and pervasiveness of measurement in so many aspects of everyday life. The study of measurement offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts, and notions of function. It highlights connections within mathematics and connections between mathematics and areas outside mathematics, such as social studies, science, art, and physical education.
- **Data Analysis, Statistics, and Probability:** Students should have experience in formulating questions, designing simple surveys and experiments, gathering and representing data, and analyzing and interpreting these data in a variety of ways. They need to explore variability by knowing and using basic measures of data spread and center, be able to describe the shape of data distributions, and be able to make inferences and draw conclusions based on information from samples of populations. They need to be able to compute probabilities of simple and compound events and to create simulations that can estimate probabilities for events.
  A natural link exists between data analysis in statistics and algebra. Students' understanding of graphs and functions can both enhance and be enhanced by tackling problems that involve data analysis and statistics in authentic situations.
- **Summary:** These guiding principles should be integral in the development of any curriculum for mathematics education. Equally important, any curriculum must be linked to assessments based on standards. A curriculum should provide a rich, connected learning experience for students while adding coherence to the standards, and standards must align with the curriculum rather than be separate, long lists of learning expectations. Alignment and coherence of these three elements—curriculum, standards, and assessment—are critically important foundations of mathematics education.

Adapted from Guiding Principles for Mathematics Curriculum and Assessment, copyright 2009 by the National Council of Teachers of Mathematics. (June 2009) Guiding Principles for Mathematics Curriculum and Assessment *http://www.nctm.org/standards/content.aspx?id=23273*

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices were developed based on the "processes and proficiencies" of two important organizations: The National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency specified in the National Research Council's report "Adding It Up."

<table>
<tr><th colspan="2">The Standards for Mathematical Practice</th></tr>
<tr><td colspan="2"><u>Practice 1</u>: Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</td></tr>
<tr><td><u>Look For's</u>

1. Students are actively engaged in solving problems.
2. Teacher provides time for and facilitates the discussion of problem situations.</td><td><u>Student Proficiencies</u>

Mathematically proficient students:
1. Understand that mathematics is relevant when studied in a cultural context.
2. Explain the meaning of a problem and restate it in their words.
3. Analyze given information to develop possible strategies for solving the problem.
4. Identify and execute appropriate strategies to solve the problem.
5. Evaluate progress toward the solution and make revisions if necessary.
6. Check their answers using a different method, and continually ask "Does this make sense?"</td></tr>
</table>

Practice 2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Look For's	**Student Proficiencies**
1. Students use varied representations and approaches when solving problems. 2. Teacher provides a range of representation of mathematical ideas and problem situations and encourages varied solution paths.	*Mathematically proficient students:* 1. Make sense of quantities and their relationships in problem situations. 2. Use varied representations and approaches when solving problems. 3. Know and flexibly use different properties of operations and objects. 4. d. Change perspectives, generate alternatives and consider different options.

Practice 3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Look For's	**Student Proficiencies**
1. Students understand and use prior learning in constructing arguments. 2. Teacher provides opportunities for students to listen to or read the conclusions and arguments of others.	*Mathematically proficient students:* 1. Understand and use prior learning in constructing arguments. 2. Habitually ask "why" and seek an answer to that question. 3. Question and problem-pose. 4. Develop questioning strategies to generate information. 5. Seek to understand alternative approaches suggested by others and, as a result, to adopt better approaches. 6. Justify their conclusions, communicate them to others, and respond to the arguments of others. 7. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

Practice 4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Look For's	Student Proficiencies
1. Students apply mathematics learned to problems they solve and reflect on results. 2. Teacher provides a variety of contexts for students to apply the mathematics learned.	*Mathematically proficient students:* 1. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians. 2. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. 3. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. 4. Analyze mathematical relationships to draw conclusions.

Practice 5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Look For's	Student Proficiencies
1. Students use technological tools to deepen understanding. 2. Teacher uses appropriate tools (e.g. manipulatives) instructionally to strengthen the development of mathematical understanding.	*Mathematically proficient students:* 1. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.) 2. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

Practice 6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Look For's	Student Proficiencies
1. Based on a problem's expectation, calculate with accuracy and efficiency. 2. Teacher emphasizes the importance of mathematical vocabulary and models precise, symbolic communication.	*Mathematically proficient students:* 1. Communicate their understanding of mathematics to others. 2. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. 3. Specify units of measure and use label parts of graphs and charts 4. Strive for accuracy.

Practice 7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Look For's	Student Proficiencies
1. Students look for, develop, and generalize arithmetic expressions, patterns, and constraints. 2. Teacher provides time for applying and discussing properties.	*Mathematically proficient students:* 1. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form. 2. Apply and discuss properties.

Practice 8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Look For's	Student Proficiencies
1. Students use repeated applications to generalize properties. 2. Teacher models and encourages students to look for and discuss regularity in reasoning.	*Mathematically proficient students:* 1. Look for mathematically sound shortcuts. 2. Use repeated applications to generalize properties.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics.

In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

STANDARDS FOR MATHEMATICAL CONTENT

The pre-kindergarten through grade 12 content standards in this framework are organized by **grade level**. Within each grade level, standards are grouped first by **domain**. Each domain is further subdivided into **clusters** of related **standards**.

- **Standards** define what students should understand and be able to do.
- **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
- **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

The table below shows which domains are addressed at each grade level:

<table>
<tr><th colspan="10">Mathematics Learning Progressions by Domain</th></tr>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">Ratios and Proportional Relationship</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="4">Statistics and Probability</td></tr>
</table>

Format for Each Grade Level

Each grade level is presented in the same format:

- an introduction and description of the critical areas for learning at that grade;
- an overview of that grade's domains and clusters; and
- the content standards for that grade (presented by domain, cluster heading, and individual standard).

ELEMENTARY
(Grades PreK-5)

MATHEMATICS PROGRAM

RECOGNITION OF AMERICAN INDIAN CULTURE AND HERITAGE IN THE CURRICULUM PROCESS

BOARD POLICY - INSTRUCTION **#2450**

The MCPS Board of Trustees fully supports Article X of the Montana Constitution and is actively committed to develop for all students an understanding of American and Montana Indian people and their histories, as well as foster respect for their respective cultures.

Because of the unique position and place in American history, the American Indian peoples' role in the development of the United States, with emphasis on the experience of the Montana Tribes, shall be included wherever appropriate in the instruction of Missoula County Public School students, in accordance with the state Constitution and state standards. Instructions concerning the historic and current roles of Indian people shall be delivered in a respectful, informative, and sensitive manner. When the social studies curriculum and other curricula are updated according to the District's curriculum cycle, the written curriculum shall reflect this policy. Staff development will be provided pertinent to curriculum implementation.

NOTE: The District has nondiscriminatory policies in effect, which may be referenced.

Legal Reference:	Art. X, Sec. 1(2), Montana Constitution	
	§§ 20-1-501, et seq., MCA	Recognition of American Indian cultural heritage - legislative intent
10.55.603 ARM	Curriculum Development and Assessment	
10.55.701 ARM	Board of Trustees	
10.55.803 ARM	Learner Access	

Policy History:

History of Previous File 2121:
- Presented to PN&P Committee for first reading, 3/30/00
- Approved First Reading, 4/11/00
- Presented to PN&P Committee for second reading, 4/27/00
- Revised at C&I Committee, 5/2/00
- Adopted on: October 10, 2000

Adopted on: January 14, 2003 (Policy recodified in Series 2000 adoption)

MONTANA OFFICE OF PUBLIC INSTRUCTION
INDIAN EDUCATION FOR ALL
ELEMENTARY LESSON PLANS
http://opi.mt.gov/Programs/IndianEd/curricsearch.html

Specific Grade Level	IEFA Math Lesson Title	URL Address
Kindergarten	Counting 1:1 Correspondence	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/GK%20Counting%201to1%20Corres.pdf
Kindergarten	Shapes in the Blackfeet Language	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/GK%20Shapes%20in%20Blackfeet.pdf
Kindergarten	Geometry and Blackfeet Portraits	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/GK%20Geometry%20and%20Blackfeet%20Portraits.pdf
Grade 1	Probability and Odds Data Analysis	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%201%20Probability%20and%20Data%20Analysis.pdf
Grade 2	Buffalo Runner	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%202%20Buffalo%20Runner.pdf
Grade 3	Pow wow Trails	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%203%20Powwow%20Trails.pdf
Grade 4	I am Beading	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%204%20I%20am%20Beading.pdf
Grade 5	Estimating the Area of a Reservation	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%205%20Estimate%20Area-Reservatn.pdf
Grade 5	Geometric Beadwork	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%205%20Geometric%20Beadwork.pdf
Grade 5	Graphing Native American Populations	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%205%20Graph%20NA%20Populations.pdf
Grade 5	Graphing Old Man's Journey	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%205%20Graph%20Old%20Man%27s%20Journey.pdf

PRE-KINDERGARTEN MATHEMATICS

Overview:

The preschool/pre-kindergarten population includes children between at least 2 years, 9 months until they are kindergarten eligible. A majority attend programs in diverse settings—community-based early care and education centers, family child care, Head Start, and public preschools. Some children do not attend any formal program. These standards apply to children who are at the end of that age group, meaning older four- and younger five-year olds.

In this age group, foundations of mathematical understanding are formed out of children's experiences with real objects and materials. The standards can be promoted through play and exploration activities, and embedded in almost all daily activities. They should not be limited to "math time." The standards should be considered guideposts to facilitate young children's underlying mathematical understanding.

In preschool or pre-kindergarten, activity time should focus on two critical areas: (1) developing an understanding of whole numbers to 10, including concepts of one-to-one correspondence, counting, cardinality (the number of items in a set), and comparison; (2) recognizing two-dimensional shapes, describing spatial relationships, and sorting and classifying objects by one or more attributes. Relatively more learning time should be devoted to developing children's sense of number as quantity than to other mathematics topics.

(1) These young children begin counting and quantifying numbers up to 10. Children begin with oral counting and recognition of numerals and word names for numbers. Experience with counting naturally leads to quantification. Children count objects and learn that the sizes, shapes, positions, or purposes of objects do not affect the total number of objects in the group. One-to-one correspondence with its matching of elements between the sets, provides the foundation for the comparison of groups and the development of comparative language such as, *more than*, *less than*, and *equal to*.

(2) Young children explore shapes and the relationships among them. They identify the attributes of different shapes including the length, area, weight by using vocabulary such as: *long*, *short*, *tall*, *heavy*, *light*, *big*, *small*, *wide*, *narrow*. They compare objects using comparative language such as: *longer/shorter*, *same length*, *heavier/lighter*. They explore and create 2- and 3-dimensional shapes by using various manipulative and play materials such as: popsicle sticks, blocks, pipe cleaners, and pattern blocks. They sort, categorize, and classify objects and identify basic 2-dimensional shapes using the appropriate language.

Counting and Cardinality

- Know number names and the counting sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.
- Work with money.

Geometry

- Identify and describe shapes (squares, circles, triangles, rectangles).
- Analyze, compare, create, and compose shapes.

Based on the Massachusetts Curriculum Framework for Mathematics, March 2011 by the Massachusetts Department of Elementary and Secondary Education.

Standards for Mathematical Practice: Pre-Kindergarten Explanations and Examples	
Standards	**Explanations and Examples**
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
PK.MP.1. Make sense of problems and persevere in solving them.	Pre-Kindergarten students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” or they may try another strategy.
PK.MP.2. Reason abstractly and quantitatively.	Pre-Kindergarten students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols.
PK.MP.3. Construct viable arguments and critique the reasoning of others.	Pre-Kindergarten students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?”
PK.MP.4. Model with mathematics.	Pre-Kindergarten students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list. Students need opportunities to connect the different representations and explain the connections.
PK.MP.5. Use appropriate tools strategically.	Pre-Kindergarten students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful.
PK.MP.6. Attend to precision.	Pre-Kindergarten students begin to develop their mathematical communication skills.
PK.MP.7. Look for and make use of structure.	Pre-Kindergarten students begin to discern a pattern or structure (i.e., abab patterns).
PK.MP.8. Look for and express regularity in repeated reasoning.	

Mathematics Standards: Counting and Cardinality

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)
Know number names and the count sequence
PK.CC.1. Listen to and say the names of numbers in meaningful contexts.
PK.CC.2. (Begins in kindergarten.)
PK.CC.3. Represent a number of objects with a written numeral 0 – 5 (with 0 representing a count of no objects).
Count to tell the number of objects
PK.CC.4. Understand the relationship between numerals and quantities to 10. a. When counting objects, say the number names in the standards order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
PK.CC.5. Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1-10, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.
Compare numbers
PK.CC.6. Identify "first" and "last" related to order or position.
PK.CC.7. (Begins in kindergarten.)

Mathematics Standards: Operations and Algebraic Thinking

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
PK.OA.1. With support and prompting, demonstrate an understanding of addition and subtraction by using objects, fingers, and responding to practical situations (e.g., if we have 3 apples and add 2 more, how many apples do we have in all?).
PK.OA.2. (Begins in kindergarten.)
PK.OA.3. (Begins in kindergarten.)
PK.OA.4. (Begins in kindergarten.)
PK.OA.5. (Begins in kindergarten).

Mathematics Standards: Number and Operations in Base Ten

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)
Work with numbers 11-19 to gain foundations for place value
PK.NBT.1. (Begins in kindergarten).

Mathematics Standards: Measurement and Data

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)
Describe and compare measurable attributes
PK.MD.1. Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., *long, short, tall, heavy, light, big, small, wide, narrow*).
Classify objects and count the number of objects in each category
PK.MD.2. Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.
PK.MD.3. Sort, categorize, and classify objects by more than one attribute.
PK.MD.4. (Begins in kindergarten.)

Mathematics Standards: Geometry

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
PK.G.1. Identify relative position of objects in space, and use appropriate language (e.g., *beside, inside, next to, close to, above, below, apart*).
PK.G.2. Identify various two-dimensional shapes regardless of their size.
PK.G.3. (Begins in kindergarten.)
Analyze, compare, create, and compose shapes
PK.G.4. Analyze, compare, and sort two- and three-dimensional shapes and objects of different sizes, using informal language to describe their similarities, differences, and other attributes (e.g., color, size, shape).
PK.G.5. Create and represent three-dimensional shapes (ball/sphere, square box/cube, tube/cylinder) using various manipulative materials (e.g., popsicle sticks, blocks, pipe cleaners, pattern blocks, clay).
PK.G.6. (Begins in kindergarten.)

KINDERGARTEN MATHEMATICS

Overview:

Domains	Counting and Cardinality	Operations and Algebraic Thinking	Number and Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Know number names and the count sequence • Counting to tell the number of objects • Compare numbers	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	• Work with numbers 11 – 19 to gain foundations for place value	• Describe and compare measurable attributes • Classify objects and count the number of objects in each category	• Identify and describe shapes • Analyze, compare, create and compose shapes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
Major Interdisciplinary Kindergarten Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Dancing With Cranes* by Ron Hall • *Good Luck Cat* by Joy Harjo • *Little Duck Sikihpsis* by Beth Cuthand	**Science** • Properties of Matter • Dinosaurs/Fossils • Observe and Describe Animals	**Social Studies Learning and Working Now and Long Ago** • Learning to Work Together • Exploring, Creating, and Communicating • Reaching Out to Times Past	

In Kindergarten, instructional time should focus on two critical areas:

1. Representing and comparing whole numbers, initially with sets of objects

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 – 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Describing shapes and space

Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

More learning time in Kindergarten should be devoted to number than to other topics.

Domain: Counting and Cardinality **K.CC**

Cluster: Know number names and the count sequence.

1. Count to 100 by ones and by tens.
   - I can count to 100 by ones.
   - I can count to 100 by tens.

2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
   - I can count forward from any given number up to 100.

3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
   - I can write my numbers from 0 to 20.
   - I can write the number that names how many objects are in a group 0 to 20.

Cluster: Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians.
   - I can count objects by touching and saying the correct number for each object.

   a. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted from a variety of cultural content, including those of Montana American Indians.
   - I can name the number of objects in a group after counting.
   - I can explain the number of objects in a group does not change even when I start counting with a different object in that group or if the group has been mixed up.

   b. Understand that each successive number name refers to a quantity that is one larger.
   - I can name the number that is one more than the group shown.
   - I can recognize a group that is one more than the group shown.

5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
   - I can count scattered objects up to groups of 10.
   - I can count organized objects that are in a group up to 20.
   - I can count out the correct amount of objects, when given a number, to make a group up to 20.

Cluster: Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Note: Include groups with up to ten objects.)
   - I can compare two groups (0 to 10) and identify which group is greater than, less than, or equal to.

7. Compare two numbers between 1 and 10 presented as written numerals.
    - I can compare two numbers from 0 to 10 and identify which is larger/smaller, more/less, greater than/less than.

Domain: Operations and Algebraic Thinking K.OA

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Note: Drawings need not show details, but should show the mathematics in the problem—this applies wherever drawings are mentioned in the Standards.
    - I can use a variety of strategies to add (including fingers, objects, pictures, sounds, etc.).
    - I can use a variety of strategies to subtract (including fingers, objects, pictures, sounds, etc.).

2. Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
    - I can solve a word problem using addition and subtraction.

3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
    - I can decompose a number from 1 to 10 and show it in different ways.

4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
    a. I can be given a number from 1 to 10, find the number to make 10, and show it in different ways.

5. Fluently add and subtract within 5.
    - I can fluently add and subtract any of the numbers 1 to 5.

Domain: Number and Operations in Base Ten K.NBT

Cluster: Work with numbers 11-19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
    - I can compose and decompose the numbers from 11 to 19 by showing how many tens and ones make a number.

Domain: Measurement and Data **K.MD**

Cluster: Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
   - I can describe different ways to measure an object.

2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
   - I can compare two objects by measurement and describe how they are different.

Cluster: Classify objects and count the number of objects in each category.

3. Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count (Note: Limit category counts to be less than equal to 10.).
   - I can sort objects into groups so that each group has something the same (color, shape, size, etc.).
   - I can count the objects in a group and put the groups in order from least to greatest.

Domain: Geometry **K.G**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects, including those of Montana American Indians, in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
   - I can identify and describe objects using names of shapes.
   - I can describe the position of an object using positional words such as, *above, below, besides, in front of, behind*, and *next to*.

2. Correctly name shapes regardless of their orientations or overall size.
   - I can name the flat/ two-dimensional and solid/ three-dimensional shapes even if they are different sizes and have been moved around. (rotated, flipped, etc.)

3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
   - I can name the flat/ two-dimensional shapes.
   - I can name the solid/three-dimensional shapes.

Cluster: Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
   - I can explain and compare the parts of a flat/two-dimensional shape.
   - I can explain and compare the parts of a solid/three-dimensional shape.

5. Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
    - I can create and draw flat/two-dimensional shapes and solid/three-dimensional shapes.

6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*
    - I can use simple shapes to compose larger shapes.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
K.MP.1. Make sense of problems and persevere in solving them.	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
K.MP.2. Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
K.MP.3. Construct viable arguments and critique the reasoning of others.	Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
K.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
K.MP.5. Use appropriate tools strategically.	Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.
K.MP.6. Attend to precision.	As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
K.MP.7. Look for and make use of structure.	Younger students begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5.
K.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). In addition, students continually check their work by asking themselves, "Does this make sense?"

Standard	Kindergarten Vocabulary (bold indicates Montana Common Core Standards vocabulary)
K.CC.1	**count**, number, sequence, ones, tens
K.CC.2	**count,** number
K.CC.3	**count,** number, object, numeral
K.CC.4	**count**, number, pair, quantity
K.CC.5	**count**, number, scattered, organized, array, line
K.CC.6	number, compare, **greater than, less than, equal to,** matching, strategies
K.CC.7	number, compare, **greater than, less than, equal to**, larger/smaller, more/less, identify
K.OA.1	**addition, add, subtraction, subtract, expression, equation**
K.OA.2	**addition, add, subtraction, subtract**, solve, word problem
K.OA.3	**decompose, equation**
K.OA.4	**add, make 10, addend, equation**
K.OA.5	**add, subtract**
K.NBT.1	**compose, decompose, equation**
K.MD.1	measure, describe, different, length, weight, height, longer, shorter, taller, heavier, lighter, attribute
K.MD.2	compare, attribute, more, less
K.MD.3	same, sort, category, classify
K.G.1	**shapes,** square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, position word, above, below, beside, in front of, behind, next to, between, under, over, by
K.G.2	**shapes,** square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, 2-dimensional, 3-dimensional
K.G.3	**shapes**, square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, 2-dimensional, 3-dimensional, solid, flat,
K.G.4	**shapes,** square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, 2**-dimensional, 3-dimensional,** corners, sides, vertex, similarities
K.G.5	**shapes,** square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, 2-dimensional, 3-dimensional
K.G.6	**shapes,** square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, 2-dimensional, 3-dimensional, **compose**

GRADE 1 MATHEMATICS

Overview:

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Represent and solve problems involving addition and subtraction • Understand and apply properties of operations and the relationship between addition and subtraction • Add and subtract within 20 • Work with addition and subtraction equations	• Extend the counting sequence • Understand place value • Use place value understanding and properties of operations to add and subtract	• Measure lengths indirectly and by iterating length units • Tell and write time • Represent and interpret data	• Reason with shapes and their attributes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Interdisciplinary Grade 1 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Two Pairs of Shoes* by Esther Sanderson • *Where did you get your Moccasins?* By Bernelda Wheeler • *White Bead Ceremony* by Sherrin Watkins	**Science** • Space: Investigating Sunlight and Moonlight • How Animals and Plants Interact in Their Environment • Nutrition/Food Pyramid	**Social Studies A Child's Place in Time and Space:** • Developing Social Skills and Responsibilities • Expanding Children's Geographic and Economic Worlds • Developing Awareness of Cultural Diversity, Now and Long Ago

In Grade 1, instructional time should focus on four critical areas:

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Developing understanding of whole number relationship and place value, including grouping in tens and ones

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Developing understanding of linear measurement and measuring lengths as iterating length units
Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Note: students should apply the principle of transitivity of measurement to make direct comparisons, but they need not use this technical term.)

4. Reasoning about attributes of, and composing and decomposing geometric shapes
Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Domain: Operations and Algebraic Thinking 1.OA

Cluster: Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
   - I can use addition and subtraction within 20 to solve word problems using objects, drawings, and equations.
   - I can solve word problems using unknowns in all positions. (8 + 2 = _, 8 + _ = 10, 10 - 8 = _, 10 - _ = 2)

2. Solve word problems within a cultural context, including those of Montana American Indians, that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
   - I can solve word problems that call for addition of three whole numbers whose sum is within 20 by using objects, drawings, and equations.
   - I can use a symbol for the unknown number.

Cluster: Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* (Note: Students need not use the formal terms for these properties.)

- I can apply strategies to add and subtract. (Examples: Commutative and Associative properties of addition, switch partners, make a ten)

4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
   - I can show the relationship between addition and subtraction. (note: subtraction is the inverse of *addition)*

Cluster: Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
   - I can use counting to add and subtract.

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
   - I can fluently add and subtract within 10.
   - I can add and subtract within 20 using a variety of strategies. (Examples: make a ten, decompose numbers, doubles or other "friendly" numbers)

Cluster: Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
   - I can interpret the meaning of the equal sign.
   - I can determine if equations are true or false.

8. Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + _ = 11, 5 = _ – 3, 6 + 6 = _.*
   - I can solve addition and subtraction equations to determine the unknown whole number.

Domain: Number and Operations in Base Ten 1.NBT

Cluster: Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
   - I can count to 120, starting from any number.
   - I can read, write, and represent numerals to 120.

Cluster: Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- I can identify how many ones are in a ten.
- I can represent two-digit numbers as tens and ones.
- I can represent numbers 11 to 19, as tens and ones.
- I can refer to multiples of ten (10, 20, 30,...) as groups of ten and 0 ones.

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
    - I can use the symbols >, <, and = to compare two two-digit numbers.

Cluster: Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
    - I can add a two-digit number and a one-digit number within 100.
    - I can add a two-digit number and multiples of ten within 100.
    - I can use a variety of strategies to show the relationship between addition and subtraction and explain my reasoning (models, drawings, place-value strategies, properties of addition and subtraction).

5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
    - I can mentally find 10 more or 10 less of a two-digit number.

6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
    - I can subtract multiples of ten from other multiples of ten within 100 using a variety of strategies to support my reasoning.

Domain: Measurement and Data 1.MD

Cluster: Measure lengths indirectly and by iterating length units.

1. Order three objects from a variety of cultural contexts, including those of Montana American Indians, by length; compare the lengths of two objects indirectly by using a third object.
    - I can order three objects by length.
    - I can compare the lengths of two objects by using the third object.

2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
   - I can measure an object using multiple shorter, same-size length units.
   - I can measure an object to the nearest whole number.

Cluster: Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.
   - I can tell time in hours and half-hours using analog and digital clocks.
   - I can write time in hours and half-hours using analog and digital clocks.

Cluster: Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
   - I can organize, represent, and interpret data with up to three categories.
   - I can ask and answer questions about data using the words total, more than, and less than.

Domain: Geometry 1.G

Cluster: Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
   - I can identify, define, draw, and create geometric shapes (square, circle, triangle, rectangle, hexagon, trapezoid, parallelogram, cube, cone, cylinder, sphere, rectangular prism).

2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Note: Students do not need to learn to formal names such as "right rectangular prism.")
   - I can compose two- and three-dimensional shapes to create new shapes.

3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
   - I can divide circles and rectangles into two or four equal shares.
   - I can use words like half, halves, fourths, quarters, and equal shares.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
1.MP.1. Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
1.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
1.MP.3. Construct viable arguments and critique the reasoning of others.	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.
1.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
1.MP.5. Use appropriate tools strategically.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.
1.MP.6. Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
1.MP.7. Look for and make use of structure.	First graders begin to discern a pattern or structure. For instance, if students recognize $12 + 3 = 15$, *then they also know* $3 + 12 = 15$. *(Commutative property of addition.) To add* $4 + 6 + 4$, *the first two numbers can be added to make a ten, so* $4 + 6 + 4 = 10 + 4 = 14$.
1.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, "Does this make sense?"

Standard	Grade 1 Montana Common Core Standards Vocabulary	Math Expressions Vocabulary
1.OA.1	addition, add, subtraction, subtract, equation, adding to, taking from, putting together, taking apart, compose, decompose	break apart
1.OA.2	addition, add, sum, equation, unknown number, symbol, solve, word problem	total, unknown partner, unknown total, story problem
1.OA.3	addition, add, subtraction, subtract, commutative property, associative property	switch partners, unknown partner, math mountain
1.OA.4	addition, subtraction, addend	partner
1.OA.5	addition, subtraction, count on, count back	count on
1.OA.6	addition, add, subtraction, subtract, count on, make a 10, fluency, decompose,	break apart, friendly numbers
1.OA.7	equal sign, equation, true, false, addition, subtraction	not equal
1.OA.8	addition, subtraction, whole number	total, unknown partner, unknown total
1.NBT.1	count, numeral	digit
1.NBT.2	two-digit number, tens, ones, bundle	teen number, decade number, ten stick, quick ten
1.NBT.3	> greater than, < less than, = equal	
1.NBT.4	add, multiple of 10, compose, place value	decade number, tens and ones, tens sticks, one circles
1.NBT.5	add, subtract, mental math,	
1.NBT.6	add, subtract, 10 more, 10 less, multiples of 10	
1.MD.1	measure, measurement, length, order, compare	standard unit of length, non-standard unit of length, estimate
1.MD.2	length, units of length, longer, shorter,	ruler, centimeters, inch
1.MD.3	time, hour, half-hour, analog clock, digital clock	half past
1.MD.4	data, organize, interpret, more, less, total	fewer, least, most, graph, table
1.G1.	square, triangle, circle, rectangle, trapezoid, hexagon, parallelogram, cube, cone, cylinder, rectangular prism, sphere, defining attributes, non-defining attributes, sides, angles, faces, sides, angles, shape	flat shapes, solid shapes
1.G.2	two-dimensional shape, three-dimensional half-circle, quarter-circle, compose, decompose, composite	combine, congruent, symmetry, symmetrical, line of symmetry
1.G.3	half, haves, half of, quarters, fourth of, quarter of, whole	

GRADE 2 MATHEMATICS

Overview:

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Represent and solve problems involving addition and subtraction • Add and subtract within 20 • Work with equal groups of objects to gain foundations for multiplication	• Understand place value • Use place value and properties of operations to add and subtract	• Measure and estimate lengths in standard units • Relate addition and subtraction to length • Work with time and money • Represent and interpret data	• Reason with shapes and their attributes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Interdisciplinary Grade 2Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Jingle Dancer* by Cynthia Leitich Smith • *Morning on the Lake* by Jan Waboose Bourdeau • *Range Eternal* by Louise Erdrich • *Red Parka Mary* by Peter Eyvindson	**Science** • States of Matter: Solids, Liquids, Gases • Life Cycles of Plants • Life Cycles of Animals	**Social Studies People Who Make a Difference:** • Parents, Grandparents, and Family Members • People Who Supply Our Needs • People from Many Cultures Now and Long Ago • Geographic Awareness

In Grade 2, instructional time should focus on four critical areas:

1. Extending understanding of base-ten notation

Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Building fluency with addition and subtraction

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. **Using standard units of measure**
   Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. **Describing and analyzing shapes**
   Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding attributes of two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Domain: Operations and Algebraic Thinking 2.OA

Cluster: Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations within a cultural context, including those of Montana American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *(Note: See Glossary, Table 1.)*
   (Whenever possible, use real world problems involving Montana American Indians)
   - I can use addition and subtraction within 100 to solve one- and two- step word problems to find an unknown number.
   - I can use drawings and equations to solve the unknown number in a problem.

Cluster: Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies. (*Note: Explanations may be supported by drawings or objects.)*
   By end of Grade 2, know from memory all sums of two one-digit numbers.
   - I can add and subtract within 20 using mental strategies.
   - I can know from memory the sum of two one-digit numbers.

Cluster: Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
   - I can determine if a group of objects (up to 20) is odd or even.
   - I can write an equation in which the sum is even using two equal addends.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
   - I can define the meaning of an array.
   - I can write an equation to represent the given array.

Domain: Number and Operations in Base Ten **2.NBT**

Cluster: Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
   - I can identify the place value of ones, tens, hundreds.
   - I can identify how many ones are in a ten.
   - I can identify how many tens are in a hundred.

2. Count within 1000; skip-count by 5s, 10s, and 100s.
   - I can count to 1000 by 5s, 10s, 100s.
   - I can skip count starting with various numbers within 100.

3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
   - I can read and write numbers to 1000 using base-ten numerals.
   - I can read and write number names to 1000.
   - I can show a number in expanded form to 1000.

4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
   - I can use >, <, and = to compare numbers

Cluster: Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
   - I can fluently add and subtract numbers to 100 without regrouping using various strategies.
   - I can fluently add and subtract numbers to 100 with regrouping using various strategies. .
   - I can show the relationship between addition and subtraction (note: subtraction is the inverse of addition).

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
   - I can add up to four two-digit numbers using various strategies.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
   - I can add and subtract to 1000 using concrete models, drawings, and strategies based on place value.
   - I can support my strategy through writing (note: journal, poster).

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
   - I can mentally add 10 or 100 to a given number 100-900.
   - I can mentally subtract 10 or 100 from a given number 100-900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations. *(Note: Explanations may be supported by drawings or objects.)*
   - I can explain why addition and subtraction strategies work using a proof drawing.

Domain: Measurement and Data 2.MD

Cluster: Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
   - I can measure objects with appropriate tools.

2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
   - I can measure objects using two different units of measurement.
   - I can relate the two measurements.

3. Estimate lengths using units of inches, feet, centimeters, and meters.
   - I can estimate lengths of objects using inches, feet, centimeters, and meters.

4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
   - I can choose a measurement tool, compare two objects and determine the difference in their lengths.

Cluster: Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems within a cultural context, including those of Montana American Indians, involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
   (Whenever possible, use real world problems involving Montana American Indians)
   - I can use addition and subtraction to solve word problems involving length.
   - I can use drawings to solve word problems involving length.
   - I can use equations to solve for an unknown number.

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
   - I can draw a number line and label whole numbers as lengths from 0-100.
   - I can find the sum or difference using my number line 0-100.

Cluster: Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
    - I can tell time to the nearest 5 minutes using analog and digital clocks.
    - I can tell the difference between a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
    - I can solve word problems using dollar bills, quarters, dimes, nickels, and pennies.
    - I can use $ (dollar) and ¢ (cent) signs properly.

Cluster: Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
    - I can gather measurements to the nearest whole unit to create data.
    - I can make a horizontal scale using measurement data.

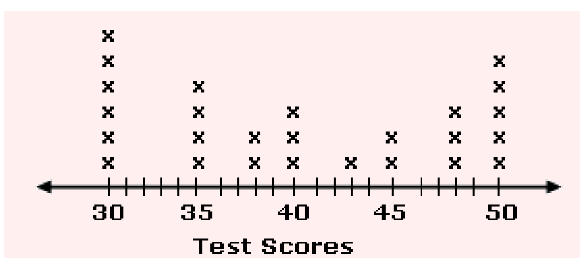
Line Plot

Definition of Line Plot

A line plot shows data on a number line with x or other marks to show frequency.

Examples of Line Plot

The line plot below shows the test scores of 26 students.



The count of cross marks above each score represents the number of students who obtained the respective score.

10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set from a variety of cultural contexts, including those of Montana American Indians, with up to four categories. Solve simple put-together, take-apart, and compare problems. *(Note: See Glossary, Table 1)* using information presented in a bar graph.
    (Whenever possible use real world problems involving Montana American Indians.)
    - I can create a picture and bar graph to represent data.
    - I can use data from graphs to solve problems (note: addition, subtraction, equal to).

Domain: Geometry **2.G**

Cluster: Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. *(Note: See Glossary, Table 1)*
   Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
   - I can identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
   - I can draw and create shapes with a given number of angles and sides.

2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
   - I can divide a rectangle into rows and columns creating same size squares. This is called finding the area.
   - I can determine the area by counting the same size squares within a given rectangle.

3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
   - I can divide circles and rectangles into 2, 3, or 4 equal parts.
   - I can use words like half, halves, thirds, fourths.
   - I can show that equal parts of identical wholes may not have the same shape.

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[1] See Glossary, Table 1.
[2] See standard 1.OA.6 for a list of mental strategies.
[3] Explanations may be supported by drawings or objects
[4] See Glossary, Table 1

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
2.MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.
2.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length.
2.MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions.
2.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using

	objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
2.MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.
2.MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
2.MP.7. Look for and make use of structure.	Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
2.MP.8. Look for and express regularity in repeated reasoning.	Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, “Does this make sense?”

Standard	Grade 2 Montana Common Core Standards Vocabulary
2.OA.1	none
2.OA.2	none
2.OA.3	odd, even
2.OA.4	rectangular array, addends
2.NBT.1	place value
2.NBT.2	none
2.NBT.3	expanded form
2.NBT.4	>, <, =
2.NBT.5	place value, commutative property, associative property, identity property
2.NBT.6	place value, commutative property, associative property, identity property
2.NBT.7	place value, commutative property, associative property, identity property, compose, decompose
2.NBT.8	none
2.NBT.9	place value, commutative property, associative property, identity property
2.MD.1	length
2.MD.2	length, unit
2.MD.3	length, unit
2.MD.4	length, unit
2.MD.5	length, unit
2.MD.6	length, unit, number line diagram, sums, differences
2.MD.7	analog clock, digital clock, a.m., p.m.
2.MD.8	dollars ($), cents (¢), quarters, dimes, nickels, and pennies
2.MD.9	length, unit, line plot, scale
2.G.1	attribute, quadrilateral, rectangle, rhombus, square, parallelogram, trapezoid, kite
2.G.2	area, unit fraction

GRADE 3 MATHEMATICS

Overview:

Domains	**Operations and Algebraic Thinking**	**Number & Operations in Base Ten**	**Number & Operations: *Fractions***	**Measurement and Data**	**Geometry**
Clusters	• Represent and solve problems involving multiplication and division • Understand properties of multiplication and the relationship between multiplication and division • Multiply and divide within 100 • Solve problems involving the four operations, and identify and explain patterns in arithmetic	• Use place value understanding and properties of operations to perform multi-digit arithmetic	• Develop understanding of fractions as numbers	• Solve problems involving measurement and estimation of intervals of time, liquid, volumes and masses of objects • Represent and interpret data • Geometric measurement: understand concepts of area and relate area to multiplication and to addition • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures	• Reason with shapes and their attributes

Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Interdisciplinary Grade 3 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Beaver Steals Fire* by Confederated Salish/Kootenai Tribes • *War Shirt* by Bently Spang • *When the Shadbush Blooms* by Carla Messinger	**Science** • Geology: Earth Materials and Changes • Weather and the Water Cycle • Simple Machines	**Social Studies Community and Change:** • Our Community and Its Heritage • Comparing Past to Present • Meeting Ordinary and Extraordinary People

In Grade 3, instructional time should focus on four critical areas (note: multiplication, division, and fractions are the most important developments):

1. Developing understanding of multiplication and division and strategies for multiplication and division within 100

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1)

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket; but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Developing understanding of the structure of rectangular arrays and of area

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. Describing and analyzing two-dimensional shapes

Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Domain: Operations and Algebraic Thinking 3.OA

Cluster: Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
    - I can explain products of whole numbers as the total number of objects in a number of groups.

2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
    - I can explain whole number quotients as the number of objects in each group when a whole number is partitioned equally.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *(Note: See Glossary, Table 2.)*
   - I can use drawings and equations to solve multiplication and division word problems involving equal groups, arrays, and measurement quantities or units of measurement.

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*
   - I can describe, using examples, that multiplication and division are inverse operations or that they are related.
   - I can solve for an unknown whole number in a multiplication and division equation.

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide. *(Note: Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
   - I can apply properties of operations as strategies to multiply and divide.

6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
   - I can solve division problems with unknown factors, using multiplication.

Cluster: Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
   - I can fluently recall multiplication and division facts within 100 using the properties of operations.
   - I can master my multiplication and division facts within 100 by the end of Grade 3.

Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. *(Note: This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.*
   - I can solve two-step word problems using the four operations.
   - I can represent an unknown quantity in an equation with a variable.

- I can decide if an answer is reasonable using estimation.

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
    - I can demonstrate my understanding of arithmetic patterns using the properties of operations.

Domain: Number and Operations in Base Ten 3.NBT

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

(Note: A range of algorithms may be used.)

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
    - I can round whole numbers to the nearest 10 or 100.

2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
    - I can fluently add and subtract within 1000.

3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
    - I can multiply one- digit whole numbers by multiples of 10 in the range 10- 90.

Number and Operations—Fractions 3.NF

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.)

Cluster: Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.
    - I can identify the parts of a fraction and explain their meanings.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
    a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
    - I can explain how a fraction is a number on a number line.
    - I can represent fractions on a number line.
    - I can divide a number line into equal intervals (parts) to represent fractions.

    b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
    - I can place fractions on a number line that is divided into equal intervals.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
      - I can show two fractions as equivalent (equal) if they are the same size.
      - I can show two fractions as equivalent (equal) if they are on the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
      - I can recognize and generate simple equivalent fractions.
      - I can justify why fractions are equivalent.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
      - I can express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
      - I can compare fractions with the same numerator.
      - I can compare fractions with the same denominator.
      - I can use >, <, = symbols to justify my conclusions when I compare fractions.

Domain: Measurement and Data 3.MD

Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
   - I can tell and write time to the nearest minute.
   - I can solve word problems involving addition and subtraction of time intervals in minutes.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Note: Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (*Note: Excludes compound units such as cm3 and finding the geometric volume of a container.)*
   - I can measure liquid volumes and masses of objects using grams, kilograms, and liters.
   - I can estimate liquid volumes and masses of objects using grams, kilograms, and liters.
   - I can add, subtract, multiply, or divide to solve word problems involving masses or volumes in the same units.

Cluster: Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories, within cultural contexts, including those of Montana American Indians. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
   - I can draw a picture graph to represent a set of data.
   - I can create a bar graph to represent a set of data.
   - I can solve one- and two- step "how many more" and "how many less" problems from a bar graph.

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
   - I can measure and record lengths to the nearest half and fourth of an inch.
   - I can use measurement data to make a horizontal line plot, which is marked off in appropriate units.

Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
      - I can use square units to measure area.
   b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
      - I can label area with square units.

6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
   - I can measure area by counting unit squares.

7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
      - I can use tiles to find the area of a rectangle.
      - I can multiply the side lengths to find the area of a rectangle.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
      - I can solve real world problems incorporating area.
   c. Products as rectangular areas in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
      - I can use tiles to make the area of a rectangle.

- I can represent the distributive property using this model.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems, including those of Montana American Indians.
   - I can find the area of irregular figures by adding the areas of smaller rectangles within the figure.
   - I can apply the area of irregular figures in real world settings.

Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
   - I can solve real world and mathematical problems involving perimeter and area of polygons.

Domain: Geometry 3.G

Cluster: Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
   - I can classify shapes by their attributes.
   - I can identify the attributes that make a rhombus, rectangle, and a square quadrilateral.
   - I can draw examples of quadrilaterals that are not rhombuses, rectangles, and squares.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*
   - I can divide shapes into equal areas.
   - I can write the area of each part of a shape as a fraction.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
3.MP.1. Make sense of problems and persevere in solving them.	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
3.MP.2. Reason abstractly and quantitatively.	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3.MP.3. Construct viable arguments and critique the reasoning of others.	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
3.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
3.MP.5. Use appropriate tools strategically.	Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
3.MP.6. Attend to precision.	As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
3.MP.7. Look for and make use of structure.	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
3.MP.8. Look for and express regularity in repeated reasoning.	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Standard	Grade 3 Montana Common Core Standards Vocabulary
3.OA.1	multiplication, factor, product
3.OA.2	division, dividend, divisor, quotient
3.OA.3	multiplication, division, array, equation
3.OA.4	multiplication, division, equation
3.OA.5	multiplication, division, commutative property, associative property, distributive property
3.OA.6	multiplication, division, factor
3.OA.7	multiplication, division, commutative property, associative property, distributive property
3.OA.8	order of operations, estimation, rounding
3.OA.9	arithmetic pattern
3.NBT.1	place value, rounding
3.NBT.2	place value, algorithm
3.NBT.3	place value, multiply
3.NF.1	fraction, unit fraction, numerator, denominator
3.NF.2	fraction, unit fraction, numerator, denominator, number line
3.NF.3	fraction, unit fraction, numerator, denominator, equivalent
3.MD.1	minute, number line
3.MD.2	volume, mass, standard units
3.MD.3	scaled picture graph, sealed bar graph
3.MD.4	line plot, scale, half/halves, quarter, fourth
3.MD.5	area, plane figure, unit square
3.MD.6	area, unit square
3.MD.7	area, area model, distributive property, additive
3.MD.8	perimeter, area
3.G.1	attribute, quadrilateral, rectangle, rhombus, square, parallelogram, trapezoid, kite
3.G.2	area, unit fraction

GRADE 4 MATHEMATICS

Overview:

Domains	**Operations and Algebraic Thinking**	**Number & Operations in Base Ten**	**Number & Operations: *Fractions***	**Measurement and Data**	**Geometry**
Clusters	• Use the four operations with whole numbers to solve problems • Gain familiarity with factors and multiples • Generate and analyze patterns	• Generalize place value understanding for multi-digit whole numbers • Use place value understanding and properties of operations to perform multi-digit arithmetic	• Extend understanding of fraction equivalence and ordering • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers • Understand decimal notation for fractions, and compare decimal fractions	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit • Represent and interpret data • Geometric measurement: understand concepts of angle and measure angles	• Draw and identify lines and angles, and classify shapes by properties of their lines and angles

Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Interdisciplinary Grade 4 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Less Than Half, More Than Whole* by Kathleen Lacapa • *Powwow* by George Ancona • *Shi-shi-etko* by Nicola L. Campbell	**Science** • Energy: Heat, Light, and Sound • Energy: Electricity and Magnetism • Local Ecosystems: Plant and Animal Interactions-Adaptations and Behavior	**Social Studies Montana and Regions of the United States:** • Learning Geography Skills • Learning About Our State and Region • Becoming Effective Citizens

In Grade 4, instructional time should focus on three critical areas:

1. Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends

Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, multiplication of fractions by whole numbers

Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry

Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Domain: Operations and Algebraic Thinking 4.OA

Cluster: Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
   - I can interpret a multiplication equation as a comparison, *e.g. Lucy has 35 cookies. Ben has 7 cookies. Lucy has 5 times as many cookies as Ben. (situation/solution equations)*
   - I can represent verbal statements of multiplication comparisons as equations.

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. *(Note: See Glossary, Table 2.)*
   - I can multiply or divide to solve word problems involving multiplicative comparison. For example, A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat costs? a * ? = p, and p / a = ?
   - I can distinguish between multiplication and addition comparison problems.

3. Solve multi-step word problems within cultural contexts, including those of Montana American Indians, with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
   - I can solve multi-step word problems using whole numbers and all four operations.
   - I can interpret a remainder as a whole number, a fraction, or a decimal.
   - I can solve these equations with an unknown variable.
   - I can check my solutions using mental math, estimation, or rounding.

Cluster: Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
    - I can identify factor pairs or multiples for all whole numbers from 1 to 100.
    - I can identify a prime or composite number 1 to 100.

Cluster: Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
    - I can create a number or shape pattern and state its rule.
    - I can explain my pattern to a given rule.

Domain: Number and Operations in Base Ten 4.NBT

Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000

Cluster: Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
    - I can recognize multiples of ten in multi-digit numbers when multiplying and dividing.

2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
    - I can read and write multi-digit whole numbers in expanded form, base ten numerals, and number names.
    - I can compare two multi- digit numbers using >, =, and < symbols (inequalities).

3. Use place value understanding to round multi-digit whole numbers to any place.
    - I can round any multi-digit number.

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
    - I can add and subtract multi-digit whole numbers using the standard algorithm.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
    - I can illustrate, multiply and explain a four-digit number by a one-digit whole number using an equation, an array, or area model.

- I can multiply, illustrate, and explain a four digit number by a two- digit whole number using an equation, an array or area model.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
   - I can divide, illustrate, and explain a division problem with a four-digit dividend by a one-digit divisor where the quotient has a remainder using an equation, an array, or area model.
   - I can recognize the distributive, associative, commutative, or identity properties, and order of operations when doing division problems.
   - I can find a quotient using an equation, array or area model.

Domain: Number and Operations—Fractions 4.NF

Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

Cluster: Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
   - I can produce equivalent fractions with visual models.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
   - I can find a common denominator.
   - I can compare fractions using >, =, < (inequalities and equalities).

Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   - I can add unit fractions.
   - I can add or subtract fractions with like denominators.

   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   - I can decompose a fraction or a mixed number into its smaller parts more than one way, e.g., ⅜ = ⅛ + ⅛ + ⅛ ; 2 ⅛ = 8/8 + 8/8 + ⅛

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- I can add and subtract improper fractions with like denominators.

d. Solve word problems within cultural contexts, including those of Montana American Indians, involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
- I can add and subtract fractions to solve word problems involving by implementing visual models and/or equations.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
- I can multiply a fraction by a whole number.
- I can demonstrate a fraction of a whole with a visual model.

b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
- I can demonstrate the associative property when multiplying a whole number by a fraction.

c. Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? As a contemporary American Indian example, for family/cultural gatherings the Canadian and Montana Cree bake bannock made from flour, salt, grease, and baking soda, in addition to ¾ cup water per pan. When making four pans, how much water will be needed?*
- I can interpret a word problem that involves multiplication of a fraction by a whole number.

Cluster: Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
- I can generate equivalent fractions where the denominators are multiples of 10.

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
   - I can convert fractions with denominators that are multiples of 10 into decimals.
   - I can compare an equivalent fraction to its corresponding decimal.

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
   - I can compare, using inequalities or equalities, two decimals to the hundredths place using a visual model.

Domain: Measurement and Data 4.MD

Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)...*
   - I can explain customary and metric units of measure.
   - I can generate a conversion table for customary and metric units of measure.
   - I can explain the units of time.

2. Use the four operations to solve word problems within cultural contexts, including those of Montana American Indians, involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
   - I can solve word problems using the four operations involving units of measurement, such as intervals of time, volume, mass, distance, and money.
   - I can create a diagram that features quantities of measurement.
   - I can show measurement in fraction and decimal form.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
   - I can apply my knowledge of area and/or perimeter to real world situations.

Cluster: Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect or arrow/spearhead collection.*

- I can create a line plot showing fractional units
- I can solve problems with a line plot showing fractional units.

Cluster: Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
   - I can recognize and measure angles.
   - I can demonstrate angles of degrees in a circle.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
   - I can interpret the measurement of an angle.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
   - I can create a specific angle using a protractor.
   - I can measure a specific angle using a protractor.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
   - I can manipulate two non-overlapping angles into a larger angle.
   - I can use addition and subtraction to find an unknown angle in a real world situation.

<u>Domain: Geometry 4.G</u>

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
   - I can create and identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
   - I can classify two-dimensional figures based on the attributes of the figures. i.e.: parallel and perpendicular lines, angles.
   - I can identify a right triangle.

3. Recognize a line of symmetry for a two-dimensional figure, including those found in Montana American Indian designs, as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
   - I can recognize lines of symmetry of two-dimensional figures.
   - I can design or draw a two-dimensional figure using lines of symmetry.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
4.MP.1. Make sense of problems and persevere in solving them.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
4.MP.2. Reason abstractly and quantitatively.	Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
4.MP.3. Construct viable arguments and critique the reasoning of others.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
4.MP.5. Use appropriate tools strategically.	Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
4.MP.6. Attend to precision.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
4.MP.7. Look for and make use of structure.	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
4.MP.8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Grade 4 Montana Common Core Standards Vocabulary		
acute angle	equivalent fraction	Order of Operations
area	estimation	parallel lines
area model	expanded form	perimeter
array	factor pair	perpendicular lines
Associative Property	factors	place value form
Commutative Property	hundredths	point
compose	Identity Property	prime number
composite number	improper fraction	product
congruent	inequalities <, =, >	quotient
customary measurement	line	rays
decimal point	line plot	remainder
decompose	line segments	right angle
degrees of an angle	mass	rounding
denominator	metric measurement	sets of
difference	mixed number	standard form
Distributive Property	multiples	sum
dividend	multiplicative comparison, i.e... as many as	symmetry
divisor	numerator	tenths
end point	obtuse angle	variables
equation	operations	volume

GRADE 5 MATHEMATICS

Overview:

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Number & Operations: *Fractions*	Measurement and Data	Geometry
Clusters	• Write and interpret numerical expressions • Analyze patterns and relationships	• Understand the place value system • Perform operations with multi-digit whole numbers and with decimals to hundredths	• Use equivalent fractions as a strategy to add and subtract fractions • Apply and extend previous understandings of multiplication and division to multiply and divide fractions	• Convert like measurement units within a given measurement system • Represent and interpret data • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	• Graph points on the coordinate plane to solve real-world and mathematical problems • Classify two-dimensional figures into categories based on their properties

Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Interdisciplinary Grade 5 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language	**Indian Education for All Titles** • *Arrow Over the Door* by Joseph Bruchac • *Navajo Long Walk* by Joseph Bruchac • *A New Look at Thanksgiving* by Catherine O'Neill Grace	**Science** • Using Variables in the Inquiry Process • Astronomy: Earth, Sun, Moon, Planets (Solar System), and Beyond • Elements and Compounds	**Social Studies United States History and Geography – Beginnings to 1850:** • Pre-Columbian America • Age of Exploration • American Indians • Settling Colonies • Causes of the American Revolution • War of Independence • Constitution • Life in the Young Republic and Westward Expansion

In Grade 5, instructional time should focus on three critical areas:

1. Developing fluency with addition and subtraction of fractions, developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)
Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operation
Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition,

subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Developing understanding of volume

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be quantified by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to solve real world and mathematical problems.

Domain: Operations and Algebraic Thinking 5.OA

Cluster: Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
    - I can evaluate expressions using parentheses, brackets, or braces.

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
    - I can write and interpret numerical expressions.

Cluster: Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
    - I can generate two different numerical patterns given two different rules.
    - I can identify and explain the relationships between the terms.
    - I can graph the ordered pairs from these terms on a coordinate plane.

Domain: Number and Operations in Base Ten 5.NBT

Cluster: Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
   - I can explain that moving one place value to the left, that digit increases by ten times the value.
   - I can explain that moving one place value to the right, the digit has 1/10 the value.

2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
   - I can explain patterns of zeros and placement of decimal points when I multiply by powers of ten.
   - I can use exponents to demonstrate the powers of ten.

3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
   - I can read, write, and compare decimals to thousandths using numerals, number names, and expanded form.

   a. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
   - I can compare two decimals to thousandths using >, =, and <.

4. Use place value understanding to round decimals to any place.
   - I can round decimals to any place.

Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
   - I can multiply multi-digit whole numbers using the standard algorithm.

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
   - I can find whole number quotients with four-digit dividends and two-digit divisors choosing from various strategies.

7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
   - I can add, subtract, multiply, and divide decimals to hundredths choosing from various strategies.
   - I can explain the reasoning behind my results.

Domain: Number and Operations—Fractions 5.NF

Cluster: Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
   - I can add and subtract fractions (including mixed numbers) with unlike denominators by finding equivalent fractions.

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
   - I can solve real-world word problems involving addition or subtractions of fractions.
   - I can use my understanding of fractions to recognize that my answer is reasonable.

Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
   - I can understand that a fraction is division of the numerator by the denominator.
   - I can solve word problems involving division of whole numbers that result in answers of fractions or mixed numbers.

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation within cultural contexts, including those of Montana American Indians. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
   - I can multiply a fraction by a whole number or another fraction.
   - I can define the components and sequence of operations to multiply fractions.

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
   - I can determine the area of a rectangle with fractional side lengths by tiling or using multiplication.

5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   - I can analyze the accuracy of a product based on comparison of the product with its factors.

   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
   - I can explain that a number multiplied by a fraction greater than one will have a greater product than that number.
   - I can explain that a number multiplied by a fraction less than one will have a smaller product than that number.
   - I can explain that a number multiplied by a fraction equal to one will stay the same (equivalent).

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians.
   - I can use fraction models or equations to solve real-world word problems.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. [1](*Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.*)
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
   - I can understand and divide a unit fraction by a whole number.

   b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*
   - I can understand and divide a whole number by a unit fraction.

   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*
   - I can apply my knowledge of dividing unit fractions by whole numbers and whole numbers by unit fractions to solve real-world problems.

Domain: Measurement and Data **5.MD**

Cluster: Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems within a cultural context, including those of Montana American Indians.
   - I can convert units of different sizes within the same system.
   - I can apply a given measurement system to solve real-world problems.

Cluster: Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
   - I can represent fractional units on a line plot.
   - I can use the data on a line plot to solve problems.

Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
   - I can define and measure volume based on a cubic unit.

   a. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
   - I can use cubic units to measure volume.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
   - I can measure volume by counting various cubic units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
   a. Within cultural contexts, including those of Montana American Indians, find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   - I can find the volume of a rectangular prism using unit cubes or multiplication.

   b. Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
   - I can apply the formula $V = l \times w \times h$ to find the volume of a rectangular prism.
   - I can apply the formula $V = b \times h$ to find the volume of a rectangular prism.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

- I can determine the volumes of two separate rectangular prisms and add them to find the total volume of the combined prisms.
- I can apply this technique to solve real-world problems.

<u>Domain: Geometry 5.G</u>

Cluster: Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
   - I can create a coordinate graph using two perpendicular lines called axes.
   - I can identify the origin as where the lines intersect and coincide with zero on each line.
   - I can plot an ordered pair in the coordinate plane.

2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation including those found in Montana American Indian designs.
   - I can represent and evaluate real-world problems by graphing in the first quadrant of the coordinate plane.

Cluster: Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
   - I can identify the attributes of all two-dimensional figures within their subcategory.

4. Classify two-dimensional figures in a hierarchy based on properties.
   - I can classify two-dimensional figures based on their properties.

Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
5.MP.1. Make sense of problems and persevere in solving them.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
5.MP.2. Reason abstractly and quantitatively.	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
5.MP.3. Construct viable arguments and critique the reasoning of others.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
5.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5.MP.5. Use appropriate tools strategically.	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
5.MP.6. Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
5.MP.7. Look for and make use of structure.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
5.MP.8. Look for and express regularity in repeated reasoning.	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Grade 5 Montana Common Core Standards Vocabulary

Operations and Algebraic Thinking	Numbers and Operations in Base Ten	Number and Operations: Fractions	Measurement & Data	Geometry
	exponents	numerator	convert	axes/axis
braces	base	denominator	conversion	intersect
brackets	powers of 10	equivalent	line plot	Origin
parentheses	digit	mixed number	unit fraction	coincide
numerical expression	number	unlike denominator	volume	coordinates
evaluate	whole number	like denominator	unit cube	coordinate system
expression	base 10 numerals	benchmark fraction	cubic unit	coordinate plane
equation	expanded form	fraction model	solid figure	ordered pair
numerical pattern	standard form	estimate	additive	x-axis
corresponding terms	number name	fraction	right rectangular prism	y-axis
ordered pairs	< less than	partition	edge	x-coordinate
coordinate plane	> greater than	unit fraction	face	y-coordinate
graph	= equal	unit cube	base	quadrant
	thousandths	scaling	vertices	point
	tenths	rectangular areas		two-dimensional figure
	hundredths	non-zero whole number		subcategory
	product	compute		category
	round			properties
	decimal			
	decimal place			
	standard algorithm			
	quotient			
	operations			
	multiplication			
	division			
	area model			
	rectangular array			

MIDDLE SCHOOL
(Grades 6-8)

MATHEMATICS PROGRAM

RECOGNITION OF AMERICAN INDIAN CULTURE AND HERITAGE IN THE CURRICULUM PROCESS

BOARD POLICY - INSTRUCTION **#2450**

The MCPS Board of Trustees fully supports Article X of the Montana Constitution and is actively committed to develop for all students an understanding of American and Montana Indian people and their histories, as well as foster respect for their respective cultures.

Because of the unique position and place in American history, the American Indian peoples' role in the development of the United States, with emphasis on the experience of the Montana Tribes, shall be included wherever appropriate in the instruction of Missoula County Public School students, in accordance with the state Constitution and state standards. Instructions concerning the historic and current roles of Indian people shall be delivered in a respectful, informative, and sensitive manner. When the social studies curriculum and other curricula are updated according to the District's curriculum cycle, the written curriculum shall reflect this policy. Staff development will be provided pertinent to curriculum implementation.
NOTE: The District has nondiscriminatory policies in effect, which may be referenced.

Legal Reference:	Art. X, Sec. 1(2), Montana Constitution	
	§§ 20-1-501, et seq., MCA	Recognition of American Indian cultural heritage - legislative intent
10.55.603 ARM	Curriculum Development and Assessment	
10.55.701 ARM	Board of Trustees	
10.55.803 ARM	Learner Access	

Policy History:
History of Previous File 2121:
Presented to PN&P Committee for first reading, 3/30/00
Approved First Reading, 4/11/00
Presented to PN&P Committee for second reading, 4/27/00
Revised at C&I Committee, 5/2/00
Adopted on: October 10, 2000

Adopted on: January 14, 2003 (Policy recodified in Series 2000 adoption)

MONTANA OFFICE OF PUBLIC INSTRUCTION
INDIAN EDUCATION FOR ALL
MIDDLE SCHOOL LESSON PLANS

http://opi.mt.gov/Programs/IndianEd/curricsearch.html

Specific Grade Level	IEFA Math LessonTitle	URL Address
Grade 6	Making a Star Quilt	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%206%20Making%20Star%20Quilt.pdf
Grade 6	Stars in the Sky	http://opi.mt.gov/PDF/IndianEd/Search/Mathematics/G%206%20Stars%20in%20the%20Sky.pdf
Grade 7	Native American Designs Power Point	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%207%20Native%20American%20Designs_ppt.pdf
Grade 7	Native American Designs Lesson Plan	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%207%20Native%20American%20Designs.pdf
Grade 8	Ko'ko'hasenestotse (Cheyenne Basket Game)	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%208%20Ko'ko'hasenestotse%20(Cheyenne%20Basket%20Game).pdf
Grade 8	Surface Area and Volume Traditional Homes	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%208%20Surface%20Area%20and%20Volume%20Traditional%20Homes.pdf

HIGH SCHOOL MATHEMATICS IN MIDDLE SCHOOL

There are some students who are able to move through mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Enrollment in both AP Statistics and AP Calculus, for example, has essentially doubled over the last decade (College Board, 2009). There is also powerful research showing that among academic factors, the strongest predictor of whether a student will earn a bachelor's degree is the highest level of mathematics taken in high school (Adelman, 1999). A recent study completed by The College Board confirms this. Using data from 65,000 students enrolled in 110 colleges, students' high school coursework was evaluated to determine which courses were closely associated with students' successful performance in college. The study confirmed the importance of a rigorous curriculum throughout a students' high school career. Among other conclusions, the study found that students who took more advanced courses, such as Pre-Calculus in the 11th grade or Calculus in 12th grade, were more successful in college. Students who took AP Calculus at any time during their high school careers were most successful (Wyatt & Wiley, 2010). And even as more students are enrolled in more demanding courses, it does not necessarily follow that there must be a corresponding decrease in engagement and success (Cooney & Bottoms, 2009, p. 2).

At the same time, there are cautionary tales of pushing underprepared students into the first course of high school mathematics in the eighth grade. The Brookings Institute's 2009 Brown Center Report on American Education found that the NAEP scores of students taking Algebra I in the eighth grade varied widely, with the bottom ten percent scoring far below grade level. And a report from the Southern Regional Education Board, which supports increasing the number of middle students taking Algebra I, found that among students in the lowest quartile on achievement tests, those enrolled in higher-level mathematics had a slightly higher failure rate than those enrolled in lower-level mathematics (Cooney & Bottoms, 2009, p. 2). In all other quartiles, students scoring similarly on achievement tests were less likely to fail if they were enrolled in more demanding courses. These two reports are reminders that, rather than skipping or rushing through content, students should have appropriate progressions of foundational content to maximize their likelihoods of success in high school mathematics.

It is also important to note that notions of what constitutes a course called "Algebra I" or "Mathematics I" vary widely. In the CCSS, students begin preparing for algebra in Kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to typical Algebra I courses—namely linear equations, inequalities, and functions—is found in the 8th grade CCSS. The Algebra I course described here ("High School Algebra I"), however, is the first formal algebra course in the Traditional Pathway (concepts from this Algebra I course are developed across the first two courses of the integrated pathway). Enrolling an eighth-grade student in a watered down version of either the Algebra I course or Mathematics I course described here may in fact do students a disservice, as mastery of algebra including attention to the Standards for Mathematical Practice is fundamental for success in further mathematics and on college entrance examinations. As mentioned above, skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later, because students may be denied the opportunity for a rigorous Algebra I or Mathematics I course and may miss important content from eighth-grade mathematics.

Middle School Acceleration

Taking the above considerations into account, as well as the recognition that there are other methods for accomplishing these goals, the Achieve Pathways Group endorses the notion that all students who are ready for rigorous high school mathematics in eighth grade should take such courses (Algebra I or Mathematics I), and that all middle schools should offer this opportunity to their students. To prepare students for high school mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. The Achieve Pathways Group has developed two compacted course sequences, one designed for districts using a traditional Algebra I – Geometry – Algebra II high school sequence, and the other for districts using an integrated sequence, which is commonly found internationally. Both are based on the idea that content should compact 3 years of content into 2 years, at most. In other words, compacting content from 2 years into 1 year would be too challenging, and compacting 4 years of content into 3 years starting in grade 7 runs the risk of compacting across middle and high schools. As such, grades 7, 8, and 9 were compacted into grades 7 and 8 (a 3:2 compaction). As a result, some 8^{th} grade content is in the 7th grade courses, and high school content is in 8th grade.

The compacted traditional sequence, or, "Accelerated Traditional," compacts grades 7, 8, and High School Algebra I into two years: "Accelerated 7th Grade" and "8th Grade Algebra I." Upon successfully completion of this pathway, students will be ready for Geometry in high school. The compacted integrated sequence, or, "Accelerated Integrated," compacts grades 7, 8, and Mathematics I into two years: "Accelerated 7th Grade" and "8th Grade Mathematics I." At the end of 8th grade, these students will be ready for Mathematics II in high school. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7^{th} Grade course to make the 8th Grade courses more manageable.

The Achieve Pathways Group has followed a set of guidelines7 for the development of these compacted courses.

1. Compacted courses should include the same Common Core State Standards as the non-compacted courses.
It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed. Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning. Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school. Traditionally, students taking high school mathematics in the eighth grade are expected to take Precalculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career (see Fourth Courses section of this paper for further detail).

Other Ways to Accelerate Students
Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra II, or Precalculus and Statistics).
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.

- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Mathematical Practices.8
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade.
- Creating a hybrid Algebra II-Precalculus course that allows students to go straight to Calculus.

A combination of these methods and our suggested compacted sequences would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career.

GRADE 6 MATHEMATICS

Overview:

Domains	**Ratios & Proportional Relationships**	**The Number System**	**Expressions and Equations**	**Geometry**	**Statistics and Probability**
Clusters	• Understand ratio concepts and use ratio reasoning to solve problems	• Apply and extend previous understandings of multiplication and division to divide fractions by fractions • Compute fluently with multi-digit numbers and find common factors and multiples • Apply and extend previous understandings of numbers to the system of rational numbers	• Apply and extend previous understandings of arithmetic to algebraic expressions • Reason about and solve one-variable equations and inequalities • Represent and analyze quantitative relationships between dependent and independent variables	• Solve real-world and mathematical problems involving area, surface area, and volume	• Develop understanding of statistical variability • Summarize and describe distributions
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
Major Thematic Grade 6 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language • Won't Grow Up - What distinguishes childhood from adulthood? • Blasts from the Past: Greek and Roman Mythology • Courageous Characters: Bravery in the Face of Danger	**Science** • Earth Process/Rocks and Minerals • Weather and Climate • Investigating Local, Regional, and Global Issues (Fire on the Lang, Montana Weed Project)	**Social Studies** • Early Humankind and the Development of Human Societies • The Beginnings of Civilization-Mesopotamia and Egypt • Review of Map Skills • The Foundation of Western Ideas-Ancient Hebrews • West Meets East-Early Civilizations of Indian and China East Meets West-Greece and Rome		

In Grade 6, instructional time should focus on four critical areas:

1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems

Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers

Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Writing, interpreting, and using expressions and equations

Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Developing understanding of statistical thinking

Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Domain: Ratios and Proportional Relationships 6.RP

Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
   - I can use ratios to describe relationships between two quantities.

2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] (Note: Expectations for unit rates in this grade are limited to non-complex fractions.)
   - I can use unit rate in the context of a ratio relationship.

3. Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   - I can make and compare tables of equivalent ratios.
   - I can plot the pairs of values from a ratio table on a coordinate plane.

   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?*
   - I can solve unit rate problems.

   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   - I can determine a percent of a quantity.

   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
   - I can use ratios to convert units of measurement.

Domain: The Number System 6.NS

Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-*

cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

- I can compute and interpret quotients of fractions and solve word problems.

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
    - I can divide multi-digit numbers.

3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
    - I can add, subtract, multiply, and divide multi-digit decimals.

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
    - I can find the greatest common factor of two whole numbers less than or equal to 100.
    - I can find the least common multiple of two whole numbers less than or equal to 12.
    - I can use the distributive property.

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
    - I can understand that positive and negative numbers have opposite values and apply it to real-world contexts.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
    - I can use a number line to recognize that numbers with opposite signs are located on opposite sides of zero.

    b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    - I can determine the location in quadrants of the coordinate plane based on the signs of the numbers in ordered pairs.
    - I can recognize ordered pairs with opposite signs are reflections of each other.

    c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- I can represent integers and other rational numbers on horizontal and vertical number line diagrams and a coordinate plane.

7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
   - I can interpret inequalities using number line diagrams.

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*
   - I can write and explain real-world statements that compare rational numbers.

   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
   - I can understand, interpret, and apply absolute value.

   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
   - I can distinguish comparisons of absolute value.

8. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
   - I can solve real-world problems using points in all four quadrants to determine distances between points.

Domain: Expressions and Equations 6.EE

Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
   - I can write and evaluate expressions with whole-number exponents.

2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
   - I can write expressions using variables.

   a. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
   - I can use mathematical terms to describe expressions.

   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*

- I can use Order of Operations to evaluate expressions.

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
   - I can apply properties of operations to generate equivalent expressions.

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
   - I can identify when two expressions are equivalent.

Cluster: Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
   - I can understand that solving an equation or inequality can be used to answer a question.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
   - I can write expressions using variables.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which *p, q* and *x* are all nonnegative rational numbers.
   - I can solve real-world problems by writing and solving equations using variables.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
   - I can write an inequality to represent a real-world situation.

Cluster: Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph

ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

- I can write an equation that shows the relationship between an independent variable and a dependent variable.
- I can analyze the relationship between dependent and independent variables using graphs and tables.

Domain: Geometry 6.G

Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area.
   - I can find the area of polygons using the area of triangles and quadrilaterals.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
   - I can find the volume of a rectangular prism.

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
   - I can use coordinate points to draw polygons on a coordinate plane.
   - I can use coordinates to find the lengths of the polygon's sides.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.
   - I can represent solids using nets made of rectangles and triangles.
   - I can determine the surface area of polygons using nets made of rectangles and triangles.

Domain: Statistics and Probability 6.SP

Cluster: Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
   - I can recognize a statistical question that anticipates variability in the data.

2. Understand that a set of data collected (including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
   - I can describe a statistical questions distribution by its center, spread, and overall shape.

3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
   - I can recognize the differences between the measures of center and the measures of variation for a numerical data set.

Cluster: Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
   - I can display data using dot plots (line plot), histograms, and box plots (box and whisker plot).

5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   - I can report the number of observations in a data set.

   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   - I can describe the attributes of a data set.

   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   - I can use measures of center and variability to describe patterns and deviations in a data set.

   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
   - I can choose which measure of center and variability best represents a data set.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
6.MP.1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
6.MP.2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
6.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
6.MP.4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
6.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6.MP.6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
6.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2 (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
6.MP.8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Standard	Grade 6 Montana Common Core Standards Vocabulary
6.RP.1	ratio
6.RP.2	ratio, rate, unit rate
6.RP.3	ratio, equivalent ratio, rate, unit rate, percent, coordinate plane
6.NS.1	quotient
6.NS.2	none
6.NS.3	none
6.NS.4	factor, multiple, GCF, LCM, distributive property
6.NS.5	positive, Negative, opposite
6.NS.6	rational number, integer, opposite, coordinate plane, ordered pair, quadrant, reflection
6.NS.7	absolute value, magnitude, rational number, positive, negative
6.NS.8	coordinate plane, quadrant, coordinates, x-coordinate, y-coordinate, absolute value
6.EE.1	base, exponent, evaluate
6.EE.2	sum, difference, term, product, factor, quotient, coefficient, arithmetic, expression, algebraic expression, substitute, evaluate
6.EE.3	equivalent expressions, commutative property, associative property, distributive property
6.EE.4	equivalent expression
6.EE.5	equation, inequality, substitute, solve, solution
6.EE.6	variable, constant, algebraic expression
6.EE.7	algebraic equation, solve
6.EE.8	inequality
6.EE.9	independent variable, dependent variable, coordinate plane
6.G.1	polygon, triangle, right triangle, quadrilateral, parallelogram, trapezoid, area, square unit
6.G.2	right rectangular prism, base, height, area, volume, cubic unit
6.G.3	vertex/vertices, coordinate, polygon
6.G.4	right rectangular prism, right triangular prism, right square pyramid, right tetrahedron, net, surface area
6.SP.1	variability
6.SP.2	distribution, center, spread, shape of data
6.SP.3	measure of center, mean, median (Q2), mode, measure of variation, range, interquartile range, extremes, lower quartile (Q1), upper quartile (Q3), outlier, mean absolute deviation
6.SP.4	line plot, dot plot, histogram, median (Q2), lower extreme, lower quartile (Q1), upper quartile (Q3), upper extreme, box plot, outlier
6.SP.5	measure of center, mean, median, mode, measure of variability, range, interquartile range, mean absolute deviation (Q2), mode, measure of variation, range, interquartile range, extremes, lower quartile (Q1), upper quartile (Q3), outlier, mean absolute deviation

GRADE 7 MATHEMATICS

Overview:

Domains	Ratios & Proportional Relationships	The Number System	Expressions and Equations	Geometry	Statistics and Probability
Clusters	• Analyze proportional relationships and use them to solve real-world and mathematical problems	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers	• Use properties of operations to generate equivalent expressions • Solve real-life and mathematical problems using numerical and algebraic expressions and equations	• Draw, construct and describe geometrical figures and describe the relationships between them • Solve real-life and mathematical problems involving angle measure, area, surface and volume	• Use random sampling to draw inferences about a population • Draw informal comparative inferences about two populations • Investigate chance processes and develop, use and evaluate probability models
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
Major Thematic Grade 7 Units	**English Language Arts: across the content areas** • Reading, • Writing • Speaking & Listening • Language • Characters with Character - What makes characters in historical fiction believable? • Perseverance - How do characters, real and fictional, use words and actions to demonstrate perseverance? • Literature Reflects Life - Is literature always a reflection of life?	**Science** • Cell Structure and Function • Energy and Life • Cell Reproduction and Genetics • Environmental Changes Through Time • Classification	**Social Studies** • Growth of Islam • African Kingdoms • Medieval China • Medieval Japan • Fall of Rome • Medieval Europe • Europe: Renaissance, Reformation, Scientific Revolution, Civilizations of the Americas		

In Grade 7, instructional time should focus on four critical areas:

1. Developing understanding of and applying proportional relationships

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Developing understanding of operations with rational numbers and working with expressions and linear equations

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Drawing inferences about populations based on samples

Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Domain: Ratios and Proportional Relationships 7.RP

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour*.
   - I can compute unit rates when given examples in various contexts.

2. Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   - I can determine if two quantities are proportional by using tables or graphs.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- I can identify and interpret the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.*

- I can develop equations to represent proportional relationships.

d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.

- I can determine unit rate given two coordinate points.
- I can explain what a point (x,y) on the graph of a proportional relationship means in in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

3. Use proportional relationships to solve multi-step ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
   - I can evaluate real world situations using multi-step ratio and percents problems within cultural contexts.

Domain: The Number System 7.NS

Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

- I can select examples to demonstrate quantities that combine to make 0 (zero).

b. Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

- I can use a number line and real-world contexts to analyze the sum of two rational numbers.
- I can justify why additive inverses equal zero.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

- I can justify, using real-world contexts, that the difference of two rational numbers is equivalent to adding the additive inverse. For example, p - q = p + (-q).
- I can show that the distance between two rational numbers on the number line is the absolute value of their differences.

d. Apply properties of operations as strategies to add and subtract rational numbers.

- I can apply properties of addition and subtraction to find sums and differences of rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

- I can interpret products of rational numbers by using properties of multiplication, particularly the distributive property.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

- I can interpret quotients of rational numbers (when the divisor is non-zero).

c. Apply properties of operations as strategies to multiply and divide rational numbers.

- I can apply properties of multiplication or division to find the product or quotient of rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

- I can convert a rational number (in a/b form) to a decimal using multiple methods.
- I can show that the decimal form of a rational number will either terminate in 0 (zero) or eventually repeats.

3. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers.

- I can decide on appropriate operations to evaluate real-world, multicultural mathematical problems involving rational numbers.

<u>Domain: Expressions and Equations 7.EE</u>

Cluster: Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
   - I can correctly apply properties of operations in order to evaluate and expand linear expressions with positive and negative coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
   - I can rewrite an equation or expression to form an equivalent equation or expression in order to shed light on the problem.

Cluster: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
    - I can evaluate a multi-step algebraic expressions and solve equations by applying the appropriate properties of mathematics and using various tools.
    - I can solve multi-step real-life mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.

4. Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    - I can construct variable equations and inequalities in order to solve multicultural real-world problems.

    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
    - I can evaluate equation word problems that compare algebraic solutions to arithmetic solutions and identify operations used.

    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
    - I can solve and graph inequalities.
    - I can analyze the solution set of an inequality.

Domain: Geometry 7.G

Cluster: Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
    - I can reproduce a geometric figure using a different scale including computing actual lengths and areas.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
   - I can construct triangles using a variety of tools, given side and/or angle measurements.
   - I can classify unique triangles by their side and/or angle measurements, and notice when conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
   - I can identify the polygon that results from a plane that cuts parallel or perpendicular to the base of a solid.

Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle.
   - I can examine the relationship (ratio) between circumference and diameter, and apply this ratio to develop formulas for area and circumference of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
   - I can apply understanding of “special angle pairs” to create and solve multi-step equations to find missing angle measures.

6. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
   - I can evaluate real-world mathematical problems involving area of polygons and surface area and volume of solids.

Domain: Statistics and Probability 7.SP

Cluster: Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
   - I can explain generalizations about a population from a sample.
   - I can justify that random sampling produces valid inferences about representative samples.

2. Use data, including Montana American Indian demographic data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled*

survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be.

- I can deduce, from multiple random samples, inferences about a population and variation in estimates.

Cluster: Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability's, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
    - I can assess the visual overlap of two data sets with similar variables and measure the mean absolute deviation of the data (For example, make comparisons between two box and whisker plots).

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
    - I can assess the measures of center and measures of variability from random samples to draw inferences about two populations.

Cluster: Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
    - I can explain the probability of an event as a number between zero and one.
    - I can evaluate if an event is likely or unlikely based on the probability written between zero and one.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.*
    - I can collect and analyze experimental probability data (especially those in a multicultural context) in order to predict future outcomes based on the relative frequency of an event.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at*

random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

- I can create a probability model where all outcomes are equally likely.
- I can create and analyze a theoretical probability model.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

- I can create an experimental probability model by observing data generated from an experiment.
- I can compare a theoretical probability model to the results of the experimental probability of that model, and explain possible sources of discrepancy.

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

- I can conclude that the probability of a compound event is the fraction of the outcome in the sample space.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

- I can create tables, tree diagrams, and organized lists for compound events.
- I can identify the outcomes in the sample space.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

- I can design a probability model to generate frequencies for compound events.

Computations with rational numbers extend the rules for manipulating fractions to complex fractions

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
7.MP.1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
7.MP.2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
7.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?". They explain their thinking to others and respond to others' thinking.
7.MP.4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
7.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
7.MP.6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
7.MP.8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Standard	Grade 7 Montana Common Core Standards Vocabulary
7.RP.1	ratio, rate, unit rate
7.RP.2	proportional relationship, constant of proportionality, unit rate, equivalent ratios, origin
7.RP.3	proportional relationship, ratio, percent
7.NS.1	Positive, negative, opposite, additive inverse, absolute value, integer, rational number
7.NS.2	integer, rational number, terminating decimal, repeating decimal
7.NS.3	rational number, complex fraction
7.EE.1	linear expression, coefficient, like terms
7.EE.2	none
7.EE.3	rational number
7.EE.4	none
7.G.1	scale drawing
7.G.2	none
7.G.3	right rectangular prism, right rectangular pyramid
7.G.4	radius, diameter, circumference, area, pi
7.G.5	supplementary angles, complementary angles, vertical angles, adjacent angles
7.G.6	length, width, base, height, altitude, area, surface area, volume
7.SP.1	sample, population, random sample, representative sample
7.SP.2	population, sample, random sample
7.SP.3	centers (also, measures of center), variabilities (also, measures of variability), mean, median, mean absolute deviation, interquartile range
7.SP.4	measures of variability, measures of center, mean, median, mean, absolute deviation, interquartile range, population, random sample
7.SP.5	likely, unlikely
7.SP.6	theoretical probability, experimental probability, relative frequency
7.SP.7	probability model, uniform probability model, frequency, relative frequency, theoretical probability, experimental probability
7.SP.8	compound events, sample space, tree diagram, outcomes, favorable outcomes, simulation

GRADE 7 MATHEMATICS - ACCELERATED

Overview:

Domains	**Ratios & Proportional Relationships**	**The Number System**	**Expressions and Equations**	**Geometry**	**Statistics and Probability**
Clusters	• Analyze proportional relationships and use them to solve real-world and mathematical problems	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers	• Use properties of operations to generate equivalent expressions • Solve real-life and mathematical problems using numerical and algebraic expressions and equations	• Draw, construct and describe geometrical figures and describe the relationships between them • Solve real-life and mathematical problems involving angle measure, area, surface and volume	• Use random sampling to draw inferences about a population • Draw informal comparative inferences about two populations • Investigate chance processes and develop, use and evaluate probability models
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
Major Thematic Grade 7 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language • Characters with Character - What makes characters in historical fiction believable? • Perseverance - How do characters, real and fictional, use words and actions to demonstrate perseverance? • Literature Reflects Life - Is literature always a reflection of life?		**Science** • Cell Structure and Function • Energy and Life • Cell Reproduction and Genetics • Environmental Changes Through Time • Classification	**Social Studies** • Growth of Islam • African Kingdoms • Medieval China • Medieval Japan • Fall of Rome • Medieval Europe • Europe: Renaissance, Reformation, Scientific Revolution, Civilizations of the Americas	

This course differs from the non-accelerated 7th Grade course in that it contains content from 8th grade. While coherence is retained, in that it logically builds from the 6th Grade, the additional content when compared to the non-accelerated course demands a faster pace for instruction and learning. Content is organized into four critical areas, or units. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

The critical areas are as follows:

Critical Area 1: Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships

between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative
numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

Critical Area 2: Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m×A. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the
properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Critical Area 3: Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Critical Area 4: Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Domain: Ratios and Proportional Relationships 7.RP

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in* *each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

- I can compute unit rates.

2. Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   - I can determine if two quantities are proportional by using tables or graphs.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   - I can interpret the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.*
   - I can develop equations to represent proportional relationships.

   d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.
   - I can determine unit rate given two coordinate points.

3. Use proportional relationships to solve multistep ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
   - I can evaluate real world situations using proportions.

Domain: The Number System 7.NS

Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   - I can describe situations in which the additive inverse has been used.

   b. Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   - I can analyze, through real-world contexts, the sum of two rational numbers.
   - I can justify why additive inverses equal zero.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- I can evaluate and apply, using real-world contexts, the difference of two rational numbers. For example, p-q = p+-q.

d. Apply properties of operations as strategies to add and subtract rational numbers.
- I can select and justify properties of addition and subtraction to find sums and differences of rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- I can interpret products of rational numbers by using properties of multiplication, particularly the distributive property.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- I can interpret quotients of rational numbers (when the divisor is non-zero).

c. Apply properties of operations as strategies to multiply and divide rational numbers.
- I can select and justify properties of multiplication and division to find the product and quotient of rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- I can convert a rational number (in a/b form) to a decimal using multiple methods.

3. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers.

a. I can choose appropriate operations to evaluate real-world mathematical problems involving rational numbers.

Domain: The Number System 8.NS.

Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
   - I can demonstrate that every number has a decimal expansion.
   - I can convert a rational number (a/b) into appropriate decimal notation.
   - I can convert a repeating decimal number into simplified rational form.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
    - I can use the appropriate estimates of irrational numbers to compare, order on a number line, and find approximate values of variable expressions.

Domain: Expressions and Equations 7.EE

Cluster: Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
    - I can correctly apply properties of operations in order to evaluate and expand linear expressions with coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
    - I can rewrite an equation or expression to form an equivalent equation or expression.

Cluster: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
    - I can evaluate multi-step algebraic expressions and equations using various tools.

4. Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    - I can construct variable equations and inequalities in order to solve multicultural real-world problems.

    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
    - I can evaluate word problems with equations that compare algebraic solutions to arithmetic solutions identifying operations used.

    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it

in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

- I can solve and graph inequalities.
- I can analyze the solution set of an inequality.

Domain: Expressions and Equations 8.EE

Cluster: Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3–5 = 3–3 = 1/33 = 1/27.
   - I can apply the properties of integer exponents to generate equivalent expressions.
2. Use square root and cube root symbols to represent solutions to equations of the form *x*2 = *p* and *x*3 = p, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
   - I can express the solution to a square root or cube root problem in radical form.
   - I can evaluate the roots of small perfect squares and cubes.
   - I can predict when a small perfect square or cube root is rational or irrational.

3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109, and determine that the world population is more than 20 times larger.*
   - I can use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or small quantities.
   - I can state how many times larger or smaller items are when quantities are in the form of a single digit times an integer power of 10.

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
   - I can perform operations in scientific notation, decimal notation, or a combination of both scientific and decimal notation.
   - I can write measurements of very large and very small quantities in scientific notation and choose units of appropriate size for the given situation.
   - I can interpret the different formats of scientific notation that have been generated by technology.

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

- I can graph proportional relationships identifying the unit rate as the slope of the graph.
- I can compare two different proportional relationships represented in different ways and state the connections between them.

6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.
   - I can use similar triangles to explain why the slope “m” is the same between any two distinct points on a non-vertical line in the coordinate plane.
   - I can derive the equation y = m* x (+0) for a line through the origin.
   - I can derive the equation y = m* x + b for a line intercepting the vertical axis at b and cannot equal 0.

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers).
   - I can solve multi-step linear equations in one variable.
   - I can solve linear equations with the same variable on both sides of the equal sign.

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
   - I can solve multi-step linear equations in one variable that include rational number coefficients, distributive property, and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   - I can graph two linear equations on the same coordinate plane and identify their point of intersection if possible.
   - I can defend that the point of intersection of two lines on the same coordinate plane is a solution for both equations.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
   - I can rewrite an equation into slope-intercept form.
   - I can solve a system of two linear equations algebraically.
   - I can estimate the solution of a system of linear equations by graphing.

   c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
   - I can solve real-world mathematical problems from a variety of cultures which involve systems of linear equations.

Domain: Geometry ***7.G***

Cluster: Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
    - I can reproduce a geometric figure using a different scale.
    - I can compute actual lengths and areas from a scale drawing.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
    - I can construct triangles using a variety of tools, given side and/or angle measurements.
    - I can classify unique triangles by their side and/or angle measurements. For example, isosceles, equilateral, scalene, obtuse, right, or acute.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
    - I can identify the polygon that results from a plane that cuts parallel or perpendicular to the base of a solid.

Domain: Geometry **8.G**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.
    - I can create and characterize reflections, rotations, and translations using a variety of tools.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
    - I can define congruency in two-dimensional figures giving examples and non-examples.
    - I can describe the sequence of transformations between two congruent figures.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians: using coordinates.
    - I can describe the effect of transformations observed in Native American geometric patterns using coordinate notation.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
    - I can define similarity in two-dimensional figures giving examples and non-examples.

- I can describe the sequence of transformations between two similar figures.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
   - I can demonstrate the sum of interior angles of any triangle is equal to 180 degrees.
   - I can generalize the patterns and relationships found between the interior and exterior angles of any triangle.
   - I can summarize the patterns and relationships found among the angle created when parallel lines are cut by a transversal.
   - I can justify similarity between triangles using angle to angle correspondence.

Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
   - I can apply the formulas for the volumes of cones, cylinders, and spheres to solve real-world mathematical problems.

Domain: Statistics and Probability 7.SP

Cluster: Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
   - I can explain generalizations about a population from a sample.
   - I can justify that random sampling produces valid inferences about representative samples.

2. Use data, including Montana American Indian demographic data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be.*
   - I can deduce, from random samples, inferences about a population and compose multiple samples to draw conclusions.

Cluster: Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability's, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability*

(mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

- I can assess the overlap of two data sets with similar variables and measure the mean absolute deviation of the data.

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
   - I can assess the measures of center and measures of variability from random samples to draw inferences about two populations.

Cluster: Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
   - I can explain the probability of an event as a number between zero and one.
   - I can evaluate if an event is likely or unlikely based on the probability written between zero and one.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.*
   - I can analyze experimental probability data in order to predict future outcomes based on the relative frequency of an event.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
   - I can create and analyze a theoretical probability model.
   - I can compare theoretical probability model to the results of the experimental probability of that model.

   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
   - I can create a probability model where all outcomes are equally likely.

   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

- I can create an experimental probability model to observe data generated from an experiment.

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
   a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
   - I can conclude that the probability of a compound event is a fraction of the outcome in the sample space.

   b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
   - I can create tables, tree diagrams, and organized lists for compound events.
   - I can identify the outcomes in the sample space.

   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
   - I can design a probability model to generate frequencies for compound events.

Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
7.MP.1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
7.MP.2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
7.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?". They explain their thinking to others and respond to others' thinking.
7.MP.4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

7.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
7.MP.6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
7.MP.8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Standard	Grade 7 Accelerated Montana Common Core Standards Vocabulary
7.RP.1	ratio, rate, unit rate
7.RP.2	proportional relationship, constant of proportionality, unit rate, equivalent ratios, origin
7.RP.3	proportional relationship, ratio, percent
7.NS.1	Positive, negative, opposite, additive inverse, absolute value, integer, rational number
7.NS.2	integer, rational number, terminating decimal, repeating decimal
7.NS.3	rational number, complex fraction
7.EE.1	linear expression, coefficient, like terms
7.EE.2	none
7.EE.3	rational number
7.EE.4	none
7.G.1	scale drawing
7.G.2	none
7.G.3	right rectangular prism, right rectangular pyramid
7.G.4	radius, diameter, circumference, area, pi
7.G.5	supplementary angles, complementary angles, vertical angles, adjacent angles
7.G.6	length, width, base, height, altitude, area, surface area, volume
7.SP.1	sample, population, random sample, representative sample
7.SP.2	population, sample, random sample
7.SP.3	centers (also, measures of center), variabilities (also, measures of variability), mean, median, mean absolute deviation, interquartile range
7.SP.4	measures of variability, measures of center, mean, median, mean, absolute deviation, interquartile range, population, random sample
7.SP.5	likely, unlikely
7.SP.6	theoretical probability, experimental probability, relative frequency
7.SP.7	probability model, uniform probability model, frequency, relative frequency, theoretical probability, experimental probability
7.SP.8	compound events, sample space, tree diagram, outcomes, favorable outcomes, simulation

GRADE 8 MATHEMATICS

Overview:

Domains	The Number System	Expressions and Equations	Functions	Geometry	Statistics & Probability
Clusters	• Know that there are numbers that are not rational, and approximate them by rational numbers	• Work with radicals and integer exponents • Understand the connections between proportional relationships, lines, and linear equations • Analyze and solve linear equations and pairs of simultaneous linear equations	• Define, evaluate, and compare functions • Use functions to model relationships between quantities	• Understand congruence and similarity using physical models, transparencies, or geometry software • Understand and apply the Pythagorean Theorem • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres	• Investigate patterns of association in bivariate data

Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Major Thematic Grade 8 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language • Figure it Out: Mysteries – What makes us want to read? • Science or Fiction – How do we determine where the line should be drawn between what we consider as fiction and what we explore as science? Does fiction fuel science or does science drive the writing of fiction? • The Road Not Taken: Going Against Conventional Wisdom – Does society always provide us with the best advice? How do we learn what to value and what choices to make? Can literature help us define the greater good?	**Science** • Structure of Matter • Properties of Matter • Basics of Energy • Forms of Energy • Forces and Motion • Simple Machines	**Social Studies** • Indigenous Cultures • Colonial Heritage • Events to the American Revolution • War for Independence • Constitution • New Nation • Age of Andrew Jackson • Regional Development • Industrial Beginnings • Pre-Civil War – Reconstruction

In Grade 8, instructional time should focus on three critical areas:

1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and *y*-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Grasping the concept of a function and using functions to describe quantitative relationships

Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem

Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Domain: The Number System **8.NS**

Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
   - I can demonstrate that every number has a decimal expansion.
   - I can convert a rational number (a/b) into appropriate decimal notation.
   - I can convert a repeating decimal number into simplified rational form.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

- I can use the appropriate estimates of irrational numbers to compare, order on a number line, and find approximate values of variable expressions.

Domain: Expressions and Equations **8.EE**

Cluster: Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
   - I can apply the properties of integer exponents to generate equivalent expressions.

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
   - I can express the solution to a square root or cube root problem in radical form.
   - I can evaluate the roots of small perfect squares and cubes.
   - I can predict when a square or cube root is rational or irrational.

3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*
   - I can use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or small quantities.
   - I can state how many times larger or smaller items are when quantities are in the form of a single digit times an integer power of 10.

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
   - I can perform operations in scientific notation, decimal notation, or a combination of both scientific and decimal notation.
   - I can write measurements of very large and very small quantities in scientific notation and choose units of appropriate size for the given situation.
   - I can interpret the different formats of scientific notation that have been generated by technology.

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
   - I can graph proportional relationships identifying the unit rate as the slope of the graph.

- I can compare two different proportional relationships represented in different ways and state the connections between them.

6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.
   - I can use similar triangles to explain why the slope "m" is the same between any two distinct points on a non-vertical line in the coordinate plane.
   - I can derive the equation y = m* x (+0) for a line through the origin.
   - I can derive the equation y = m* x + b for a line intercepting the vertical axis at b and cannot equal 0.

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers).
   - I can solve multi-step linear equations in one variable.
   - I can solve linear equations with the same variable on both sides of the equal sign.

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
   - I can solve multi-step linear equations in one variable that include rational number coefficients, distributive property, and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   - I can graph two linear equations on the same coordinate plane and identify their point of intersection if possible.
   - I can verify and defend that the point of intersection of two lines on the same coordinate plane is a solution for both equations.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
   - I can rewrite an equation from standard form into slope-intercept form.
   - I can solve a system of two linear equations algebraically.
   - I can estimate the solution of a system of linear equations by graphing.

   c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
   - I can solve real-world mathematical problems from a variety of cultures which involve systems of linear equations.

Domain: Functions **8.F**

Cluster: Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
   ([1]Function notation is not required in Grade 8.)
   - I can define a function as a rule for ordered pairs that shows each input has exactly one output.
   - I can relate input to output in graphical form as ordered pairs.

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
   - I can create a function table.
   - I can graph the contents of a function table.
   - I can write a function rule in y = m * x + b form from multiple sources.
   - I can compare and analyze two functions represented in different forms.

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
   - I can identify the attributes of linear or non-linear functions based on multiple sources.

Cluster: Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
   - I can determine the rate of change and initial value from a table, a graph, an equation, and a verbal model.
   - I can write a function rule (y = m * x + b) from any of the other three representations.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
   - I can write a verbal model of a graph showing a functional relationship.
   - I can produce an approximate graph of a functional relationship from a verbal model.

Domain: Geometry **8.G**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians:
   a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.
- I can create and characterize reflections, rotations, and translations using a variety of tools.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
   - I can define congruency in two-dimensional figures giving examples and non-examples.
   - I can describe the sequence of transformations between two congruent figures.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians: using coordinates.
   - I can describe the effect of transformations observed in Native American geometric patterns using coordinate notation.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
   - I can define similarity in two-dimensional figures giving examples and non-examples.
   - I can describe the sequence of transformations between two similar figures.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
   - I can demonstrate the sum of interior angles of any triangle is equal to 180 degrees.
   - I can generalize the patterns and relationships found between the interior and exterior angles of any triangle.
   - I can summarize the patterns and relationships found among the angles created when parallel lines are cut by a transversal.
   - I can justify similarity between triangles using angle to angle correspondence.

Cluster: Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
   - I can explain and prove the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *For example, determine the unknown height of a Plains Indian tipi when given the side length and radius.*
   - I can apply the Pythagorean Theorem to determine unknown side lengths in 2D and 3D real world situations.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
   - I can apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
   - I can apply the formulas for the volumes of cones, cylinders, and spheres to solve real-world mathematical problems.

Domain: Statistics and Probability 8.SP

Cluster: Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
   - I can define and create examples of clustering, outliers, positive or negative association, linear association, and nonlinear association.
   - I can construct and interpret scatter plots to investigate patterns of association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
   - I can sketch a line of best fit for a graph of bivariate data.(scatter plot)
   - I can construct and interpret scatter plots to investigate patterns of association.
   - I can use the closeness of the data points to the line of best fit to assess the correlation between the predicted values and the actual data.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
   - I can interpret the slope and intercept of a line of best fit in the context of the bivariate data set.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
   - I can construct a two-way frequency table from a variety of cultural contexts, including data from Montana American Indian sources.

- I can interpret relative frequencies calculated for rows or columns to describe possible associations between the two variables.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
8.MP.1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
8.MP.2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
8.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
8.MP.4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
8.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
8.MP.6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
8.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8.MP.8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 8 Montana Common Core Vocabulary									
Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary
S.ID.2	absolute deviation	A.REI.3	equation	8.G.6	leg	8.G.6	Pythagorean Theorem	A.CED.3	union
F.IF.7	absolute value	A.CED.4	equivalent equation	A.REI.3	like terms	F.LE.1	quadratic equation	F.LE.1	unit rate
F.IF.7	absolute value equation	A.CED.4	equivalent expression	8.SP.2	line of best fit/trend line	A.CED.1	quadratic formula	8.SP.4	univariate
F.IF.7	absolute value function	A.REI.5	equivalent inequalities	A.REI.3	linear	F.LE.1	quadratic function	A.CED.3	universal set
A.RIE.3	additive identity	F.LE.1	exponent	8.SP.1	linear association	A.REI.2	radical	A.CED.1	variable
A.REI.3	additive inverse	A.CED.1	exponential	A.CED.1	linear equation	A.REI.2	radical expression	F.LE.1	vertex
F.LE.2	arithmetic sequence	F.IF.8	exponential decay	A.CED.2	linear function	A.REI.2	radical function	F.LE.1	vertex form of a quadratic equation
F.IF.7	axis of symmetry	F.LE.1	exponential function	A.CED.1	linear inequality	A.REI.2	radical symbol	F.LE.3	vertical motion model
A.CED.1	base	F.IF.8	exponential growth	F.IF.7	linear model	F.IF.1	range	A.REI.1	x and y intercepts
A.REI.4	binomial	A.SSE.1	expression	F.IF.8	linear regression	F.IF.6	rate	A.SSE.3	zero exponent
8.SP. 4	bivariate	A.REI.2	extraneous solution	F.IF.7	linear representation	F.IF.6	rate of change	A.SSE.3	zeros of a function
S.ID.1	box and whisker plot	F.BF.1	extrapolation	A.CED.4	literal equation	F.IF.6	ratio		
S.ID.9	causation	S.ID.3	extreme value	A.SSE.3	maximum value/maxima	N.RN.1	rational equation		
A.REI.2	closed system	A.SSE.1	factor	S.ID.3	measures of central tendency	N.RN.1	rational number		
8.SP.1	clustering	A.SSE.3	factor completely	A.SSE.3	minimum value/minima	N.RN.3	real number		
A.SSE.1	coefficient	A.REI.4	factoring	A.APR.1	monomial	A.REI.3	reciprocal		
A.REI.4	completing the square	F.IF.8	family of function	A.REI.3	multiplicative identity	F.BF.1	recursive		
A.CED.3	compound inequality	S.ID.5	frequency	A.REI.3	multiplicative inverse	A.CED.1	relation		
A.REI.4	compound interest	A.CED.1	function	8.SP.1	negative association/correlation	A.REI.2	restricted domain		
A.REI.3	consistent dependent system	A.REI.1	function notation	A.SSE.3	negative exponent	A.SSE.3	roots		
A.REI.3	consistent independent system	F.LE.2	geometric sequence	8.SP.1	no correlation	F.IF.7	scale		
F.IF.6	constant of variation	A.CED.2	graph ordered pairs	8.SP.1	nonlinear	8.SP.1	scatter plot		
A.REI.3	constant term	F.IF.8	growth factor	8.SP.1	nonlinear association	A.CED.3	set		
A.CED.3	constraints	F.IF.8	growth rate	F.IF.8	order of mag	F.IF.6	slope		
8.G.6	converse	S.ID.1	histogram	A.CED.2	ordered pair	F.IF.7	slope-intercept form		
A.CED.2	coordinate plane	8.G.6	hypotenuse	F.IF.7	origin	A.REI.3	solution		
S.ID.9	correlation	A.REI.5	identity	8.F.1	output	A.REI.5	solution to a system		
S.ID.8	correlation coefficient	A.REI.3	inconsistent system	8.SP.1	outlier	A.REI.5	solution to inequality		
N.RN.1	cube root	8.F.1	independent variable	F.IF.7	parabola	N.RN.1	square root		
A.REI.4	decay factor	A.CED.1	inequality	F.IF.8	parent function	A.REI.2	square root function		
A.APR.1	degree of polynomial	8.F.1	input	F.IF.8	parent quadratic function	S.ID.2	standard deviation		
8.F.1	dependent variable	N.RN.1	integer	N.RN.1	perfect square	A.REI.10	standard form		

Grade 8 Montana Common Core Vocabulary									
Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary
A.CED.1	direct variation	N.RN.1	integer exponent	A.SSE.3	perfect square trinomial	F.LE.1	standard form of a quadratic function		
A.REI.4	discriminant	F.BF.1	interpolation	F.IF.4	periodicity	F.IF.7	step function		
A.REI.3	distributive property	S.ID.2	interquartile range	F.IF.7	piecewise function	8.SP.1	strong correlation		
F.IF.1	domain	8.EE.8	intersection	F.IF.7	point slope form	F.IF.4	symmetry		
S.ID.1	dot plot	F.BF.4	inverse function	A.APR.1	polynomials	A.REI.12	system of linear inequalities		
A.CED.3	element	A.REI.3	inverse operations	8.SP.1	positive association/correlation	8.EE.8	system of linear equations		
A.CED.3	empty set	N.RN.1	irrational number	F.LE.1	power	A.REI.3	term		
F.LE.1	equal intervals	A.SSE.2	leading coefficient	F.IF.8	properties of exponents	A.APR.1	trinomial		

GRADE 8 MATHEMATICS
ACCELERATED STRAND: ALGEBRA 1

Overview:

Domains	**Seeing Structure in Expressions**	**Arithmetic with Polynomials and Rational Functions**	**Creating Equations**	**Reasoning with Equations and Inequalities**
Clusters	• Interpret the structure of expressions • Write Expressions in equivalent forms to solve problems	• Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	• Create equations that describe numbers or relationships	• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Major Thematic Grade 8 Units	**English Language Arts: across the content areas** • Reading • Writing • Speaking & Listening • Language • Figure it Out: Mysteries – What makes us want to read? • Science or Fiction – How do we determine where the line should be drawn between what we consider as fiction and what we explore as science? Does fiction fuel science or does science drive the writing of fiction? • The Road Not Taken: Going Against Conventional Wisdom – Does society always provide us with the best advice? How do we learn what to value and what choices to make? Can literature help us define the greater good?		**Science** • Structure of Matter • Properties of Matter • Basics of Energy • Forms of Energy • Forces and Motion • Simple Machines	**Social Studies** • Indigenous Cultures • Colonial Heritage • Events to the American Revolution • War for Independence • Constitution • New Nation • Age of Andrew Jackson • Regional Development • Industrial Beginnings • Pre-Civil War – Reconstruction

The fundamental purpose of this accelerated 8th Grade course is to formalize and extend the mathematics that students learned through the end of seventh grade. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. In addition, the units will introduce methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course differs from High School Algebra 1 in that it contains content from 8th grade. While coherence is retained, in that it logically builds from the Accelerated 7th Grade, the additional content when compared to the high school course demands a faster pace for instruction and learning.

Critical Area 1: Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Domain: Expressions and Equations 8.EE

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

8. Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

- I can graph two linear equations on the same coordinate plane and identify their point of intersection if possible.
- I can defend that the point of intersection of two lines on the same coordinate plane is a solution for both equations.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

- I can rewrite an equation from standard form into slope-intercept form.
- I can solve a system of two linear equations algebraically.
- I can estimate the solution of a system of linear equations by graphing.

c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

- I can solve real-world mathematical problems from a variety of cultures which involve systems of linear equations.

Domain: Functions 8.F

Cluster: Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
([1]Function notation is not required in Grade 8.)

- I can define a function as a rule for ordered pairs that shows each input has exactly one output.
- I can relate input to output in graphical form as ordered pairs.

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

- I can create a function table.
- I can graph the contents of a function table.

- I can write a function rule in y = m * x + b form from multiple sources.
- I can compare and analyze two functions represented in different forms.

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
   - I can identify the attributes of linear or non-linear functions based on multiple sources.

Cluster: Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
   - I can determine the rate of change and initial value from a table, a graph, an equation, and a verbal model.
   - I can write a function rule (y = m * x + b) from any of the other three representations.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
   - I can write a verbal model of a graph showing a functional relationship.
   - I can produce an approximate graph of a functional relationship from a verbal model.

Domain: Geometry 8.G

Cluster: Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
   - I can explain and prove the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *For example, determine the unknown height of a Plains Indian tipi when given the side length and radius.*
   - I can apply the Pythagorean Theorem to determine unknown side lengths in 2D and 3D real world situations.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
   - I can apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Domain: Statistics and Probability 8.SP

Cluster: Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

- I can define and create examples of clustering, outliers, positive or negative association, linear association, and nonlinear association.
- I can construct and interpret scatter plots to investigate patterns of association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
   - I can sketch a line of best fit for a graph of bivariate data.(scatter plot)
   - I can construct and interpret scatter plots to investigate patterns of association.
   - I can use the closeness of the data points to the line of best fit to assess the correlation between the predicted values and the actual data.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
   - I can interpret the slope and intercept of a line of best fit in the context of the bivariate data set.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
   - I can construct a two-way frequency table from a variety of cultural contexts, including data from Montana American Indian sources.
   - I can interpret relative frequencies calculated for rows or columns to describe possible associations between the two variables.

Number and Quantity Content Standards

Domain: The Real Number System **N-RN**

Cluster: Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
   - I can apply the properties of exponent to rational exponents.
   - I can explain how rational exponents follow from the properties of integer exponents. (See above)

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- I can write radical expressions using rational exponents and vice versa.

Cluster: Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
   - I can use the closure property or show by example the sum or product or two rational numbers are rational.
   - I can use the closure property or show by example the sum of a rational and an irrational number is irrational.
   - I can use the closure property or show by example the product or a nonzero rational number and an irrational is irrational.

<u>Domain: Quantities N-Q</u>

Cluster: Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
   - I can interpret units in the context of the problem.
   - I can use unit analysis to check the reasonableness of my solution.
   - I can choose and interpret an appropriate scale given data to be represented on a graph or display.

2. Define appropriate quantities for the purpose of descriptive modeling.
   - I can determine an appropriate quantity to model a situation.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
   - I can choose a level of accuracy appropriate to the measuring tool or situation.

Algebra Content Standards

<u>Domain: Seeing Structure in Expressions A-SSE</u>

Cluster: Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   - I can interpret expressions that represent a quantity in terms of its context.
   - I can identify the different parts of an expression and explain their meaning within the context of a problem.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
   - I can interpret expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$*.*
- I can rewrite algebraic expressions in equivalent forms such as factored or simplified form.
- I can use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor an expression completely.
- I can simplify expressions by combining like terms, using the distributive property and using other operations with polynomials.

Cluster: Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   - I can choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   - I can write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros.

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   - I can complete the square in a quadratic expression to convey the vertex form and determine the maximum or minimum value of the quadratic function, and to explain the meaning of the vertex.

   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
   - I can use properties of exponents (such as power of a power, product of powers, power of a product, power of a quotient) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

Domain: Arithmetic with Polynomials and Rational Expressions A-APR

Cluster: Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
   - I can identify polynomials.
   - I can add, subtract, and multiply polynomials.
   - I can recognize how closure applies under these operations.

Domain: Creating Equations A-CED

Cluster: Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

- I can create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
   - I can create equations in two or more variables to represent relationships between quantities.
   - I can graph equations in two variables on a coordinate plane and label the axes and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
   - I can write and use a system of equations and/or inequalities to solve a real world problem.
   - I can use equations and inequalities to represent problem constraints and objectives (linear programming).

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
   - I can solve multi-variable formulas or literal equations for a specific variable.

Domain: Reasoning with Equations and Inequalities A-REI

Cluster: Understand solving equations as a process of reasoning and explain reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
   - I can construct a convincing argument that justifies each step in the solution process assuming an equation has a solution.

Cluster: Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
   - I can solve linear equations in one variable, including equations with coefficients represented by letters.
   - I can solve linear inequalities in one variable, including inequalities with coefficients represented by letters.

4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)2 = q$ that has the same solutions. Drive the quadratic formula from this form.
   - I can solve quadratic equations in one variable.
   - I can transform a quadratic equation to an equation in the form $(x – p2 = q$ by completing the square.
   - I can drive the quadratic formula by completing the square on a quadratic equation.

b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

- I can solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square.
- I can explain why taking the square root of both sides of an equation can yield two solutions.
- I can use the quadratic formula to solve quadratic equation, recognizing the formula produces all complex solutions and write the solutions in the form $a \pm bi$ where a and b are real numbers.

Cluster: Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
   - I can produce, with justification, from a system of two equations an equivalent simpler system.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
   - I can solve systems of equations using substitution, linear combination, and graphing.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x2 + y2 = 3$.
   - I can solve a system containing a linear equation and a quadratic equation in two variables algebraically and graphically.

Cluster: Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
    - I can find any solution to an equation in two variables from the graph of that equation.

11. Explain why the *x*-coordinates of the points where the graphs of the equations $y – f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential.
    - I can explain why the intersection of $y = f(x)$ and $y = g(x)$ is the solution of $f(x) = g(x)$ for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
    - I can use technology to graph the equations and find their points of intersection.
    - I can use tables of values or successive approximations to find solutions.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

- I can graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for strict inequalities.

Functions Content Standards

Domain: Interpreting Functions **F-IF**

Cluster: Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
   - I can use the definition of a function to determine whether a relationship is a function given a table, graph or words.
   - I can identify x as an element of the domain and $f(x)$ as an element in the range given the function f.
   - I can identify that the graph of the function f is the graph of the function $y=f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
   - I can use function notation, $f(x)$, when a relation is determined to be a function.
   - I can evaluate functions for inputs in their domains.
   - I can interpret statements that use function notation in terms of a context in which they are used.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
   - I can recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
   - I can write a recursive formula in function notation for a generated sequence.

Cluster: Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
   - I can identify key features in graphs and tables to include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior for a linear, exponential and quadratic function.
   - I can sketch the graph of a function given its key features.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- I can interpret a graph to determine the appropriate numerical domain being described in the linear, exponential and quadratic functions.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.⋆
   - I can calculate and interpret the average rate of change of a function presented symbolically or as a table.
   - I can estimate the average rate of change over a specified interval of a function from its graph.

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   - I can graph linear functions showing intercepts.
   - I can graph quadratic functions showing intercepts, a maximum or a minimum.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   - I can graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions.

   e. Graph exponential showing intercepts.
   - I can graph exponential functions, showing intercepts and end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   - I can use the process of factoring and completing the square in a quadratic function to show zeros, a maximum or minimum, and symmetry of the graph, and interpret these in terms of a real-world situation.
   - I can explain different properties of a function that are revealed by writing a function in equivalent forms.

   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
   - I can use the properties of exponents to interpret exponential functions as growth or decay.
   - I can identify the percent rate of change in an exponential function.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
   - I can compare the key features of two linear, exponential, quadratic, absolute value, step and piecewise defined functions that are represented in different ways.

<u>Domain: Building Functions F-BF</u>

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   - I can write an explicit or recursive expression or describe the calculations needed to model a function given a situation.
   - I can write a linear, quadratic or exponential function that describes a relationship between two quantities.

   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   - I can combine function types, such as linear and exponential, using arithmetic operations.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.⋆
   - I can make connections between linear functions and arithmetic sequences, and exponential functions and geometric sequences.
   - I can write and translate between the recursive and explicit formula for a arithmetic sequence and use the formulas to model a situation.
   - I can write and translate between the recursive and explicit formula for a geometric sequence and use the formulas to model a situation

Cluster: Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
   - I can experiment to identify, using technology, the transformational effects on the graph of a function $f(x)$ (*linear, exponential, quadratic or absolute value functions*) when $f(x)$ is replaced by $f(x)+k$, $k{\cdot}f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k, both positive and negative.
   - I can find the value of k given the graph of a transformed function.
   - I can recognize even and odd functions from their graphs and equations.

4. Find inverse functions.
   a. Solve an equation of the form $f(x) = ax + b$ for a simple function f that has an inverse and write an expression for the inverse. *For linear functions only.*
   - I can solve a linear function for the dependent variable and write the inverse of a linear function by interchanging the dependent and independent variables.

Domain: Linear, Quadratic, and Exponential Models F-LE

Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   - I can determine a situation as linear or exponential by examining rates of change between data points.
   - I can show there is a constant difference in a linear function over equal intervals.
   - I can show there is a constant ratio in an exponential function over equal intervals.

   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   - I can describe situations where one quantity grows or decays by a constant ratio per unit interval relative to another.

   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
   - I can describe situations where one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
   - I can write a linear or exponential function given an arithmetic or geometric sequence, a graph, a description of the relationship, or two points which can be read from a table.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
   - I can use graphs and tables to make the connection that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or any other polynomial function.

Cluster: Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.
   - I can explain the meaning of the coefficients, constants, factors, exponents, and intercepts in a linear or exponential function in terms of a context.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Statistics and Probability Content Standards

Domain: Interpreting Categorical and Quantitative Data S-ID

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
   - I can construct dot plots, histograms and box plots on a real number line.

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
   - I can describe a distribution using center and spread.
   - I can use the correct measure of center and spread to describe a distribution that is symmetric or skewed.
   - I can compare two or more different data sets using the center and spread of each.

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
   - I can identify outliers (extreme data points) using IQR and their effects on data sets.
   - I can interpret differences in different data sets in context.
   - I can interpret differences due to possible effects of outliers.

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
   - I can create a two-way table from two categorical variables and read values from a two-way table
   - I can interpret joint, marginal, and relative frequencies in context.
   - I can recognize associations and trends in data from a two-way table.

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear models. Discuss general principles referring to quadratic, and exponential models.
   - I can create a scatter plot from two quantitative variables.
   - I can describe the form (linear, quadratic or exponential), strength (strong to weak) and
     1. direction (positive or negative) of the relationship.
   - I can explain the meaning of slope and y-intercept (linear model) or the meaning of the growth rate and y-intercept (exponential model) or the meaning of the coefficients (quadratic model) in context.

- I can use algebraic methods or technology to fit the data to a linear, exponential or quadratic function.

b. Informally assess the fit of a function by plotting and analyzing residuals.
- I can calculate a residual.
- I can create and analyze a residual plot.

c. Fit a linear function for a scatter plot that suggests a linear association.
- I can use algebraic methods or technology to fit the data to a linear function.
- I can use the function to predict values.

Cluster: Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
   - I can explain the meaning of the slope and y-intercept in context.

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
   - I can use a calculator or computer to find the correlation coefficient for a linear association.
   - I can interpret the meaning of the correlation coefficient in the context of the data.

9. Distinguish between correlation and causation.
   - I can explain the difference between correlation and causation.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
8.MP.1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
8.MP.2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
8.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
8.MP.4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
8.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
8.MP.6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
8.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8.MP.8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 8 Accelerated Montana Common Core Vocabulary

Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary
S.ID.2	absolute deviation	A.REI.3	equation	8.G.6	leg	8.G.6	Pythagorean Theorem	S.ID.5	two way table
F.IF.7	absolute value	A.CED.4	equivalent equation	A.REI.3	like terms	F.LE.1	quadratic equation	A.CED.3	union
F.IF.7	absolute value equation	A.CED.4	equivalent expression	8.SP.2	line of best fit/trend line	A.CED.1	quadratic formula	F.LE.1	unit rate
F.IF.7	absolute value function	A.REI.5	equivalent inequalities	A.REI.3	linear	F.LE.1	quadratic function	8.SP.4	univariate
A.RIE.3	additive identity	F.LE.1	exponent	8.SP.1	linear association	A.REI.2	radical	A.CED.3	universal set
A.REI.3	additive inverse	A.CED.1	exponential	A.CED.1	linear equation	A.REI.2	radical expression	A.CED.1	variable
F.LE.2	arithmetic sequence	F.IF.8	exponential decay	A.CED.2	linear function	A.REI.2	radical function	F.LE.1	vertex
F.IF.7	axis of symmetry	F.LE.1	exponential function	A.CED.1	linear inequality	A.REI.2	radical symbol	F.LE.1	vertex form of a quadratic equation
A.CED.1	base	F.IF.8	exponential growth	F.IF.7	linear model	F.IF.1	range	F.LE.3	vertical motion model
A.REI.4	binomial	A.SSE.1	expression	F.IF.8	linear regression	F.IF.6	rate	A.REI.1	x and y intercepts
8.SP. 4	bivariate	A.REI.2	extraneous solution	F.IF.7	linear representation	F.IF.6	rate of change	A.SSE.3	zero exponent
S.ID.1	box and whisker plot	F.BF.1	extrapolation	A.CED.4	literal equation	F.IF.6	ratio	A.SSE.3	zeros of a function
S.ID.9	causation	S.ID.3	extreme value	A.SSE.3	maximum value/maxima	N.RN.1	rational equation		
A.REI.2	closed system	A.SSE.1	factor	S.ID.3	measures of central tendency	N.RN.1	rational number		
8.SP.1	clustering	A.SSE.3	factor completely	A.SSE.3	minimum value/minima	N.RN.3	real number		
A.SSE.1	coefficient	A.REI.4	factoring	A.APR.1	monomial	A.REI.3	reciprocal		
A.REI.4	completing the square	F.IF.8	family of function	A.REI.3	multiplicative identity	F.BF.1	recursive		
A.CED.3	compound inequality	S.ID.5	frequency	A.REI.3	multiplicative inverse	A.CED.1	relation		
A.REI.4	compound interest	A.CED.1	function	8.SP.1	negative association/correlation	A.REI.2	restricted domain		
A.REI.3	consistent dependent system	A.REI.1	function notation	A.SSE.3	negative exponent	A.SSE.3	roots		
A.REI.3	consistent independent system	F.LE.2	geometric sequence	8.SP.1	no correlation	F.IF.7	scale		
F.IF.6	constant of variation	A.CED.2	graph ordered pairs	8.SP.1	nonlinear	8.SP.1	scatter plot		
A.REI.3	constant term	F.IF.8	growth factor	8.SP.1	nonlinear association	A.CED.3	set		
A.CED.3	constraints	F.IF.8	growth rate	F.IF.8	order of mag	F.IF.6	slope		
8.G.6	converse	S.ID.1	histogram	A.CED.2	ordered pair	F.IF.7	slope-intercept form		
A.CED.2	coordinate plane	8.G.6	hypotenuse	F.IF.7	origin	A.REI.3	solution		
S.ID.9	correlation	A.REI.5	identity	8.F.1	output	A.REI.5	solution to a system		
S.ID.8	correlation coefficient	A.REI.3	inconsistent system	8.SP.1	outlier	A.REI.5	solution to inequality		
N.RN.1	cube root	8.F.1	independent variable	F.IF.7	parabola	N.RN.1	square root		
A.REI.4	decay factor	A.CED.1	inequality	F.IF.8	parent function	A.REI.2	square root function		
A.APR.1	degree of polynomial	8.F.1	input	F.IF.8	parent quadratic function	S.ID.2	standard deviation		
8.F.1	dependent variable	N.RN.1	integer	N.RN.1	perfect square	A.REI.10	standard form		
A.CED.1	direct variation	N.RN.1	integer exponent	A.SSE.3	perfect square trinomial	F.LE.1	standard form of a quadratic function		

Grade 8 Accelerated Montana Common Core Vocabulary

Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary	Standard	Vocabulary
A.REI.4	discriminant	F.BF.1	interpolation	F.IF.4	periodicity	F.IF.7	step function		
A.REI.3	distributive property	S.ID.2	interquartile range	F.IF.7	piecewise function	8.SP.1	strong correlation		
F.IF.1	domain	8.EE.8	intersection	F.IF.7	point slope form	F.IF.4	symmetry		
S.ID.1	dot plot	F.BF.4	inverse function	A.APR.1	polynomials	A.REI.12	system of linear inequalities		
A.CED.3	element	A.REI.3	inverse operations	8.SP.1	positive association/correlation	8.EE.8	system of linear equations		
A.CED.3	empty set	N.RN.1	irrational number	F.LE.1	power	A.REI.3	term		
F.LE.1	equal intervals	A.SSE.2	leading coefficient	F.IF.8	properties of exponents	A.APR.1	trinomial		

HIGH SCHOOL (Grades 9-12)

MATHEMATICS PROGRAM

RECOGNITION OF AMERICAN INDIAN CULTURE AND HERITAGE IN THE CURRICULUM PROCESS

BOARD POLICY - INSTRUCTION **#2450**

The MCPS Board of Trustees fully supports Article X of the Montana Constitution and is actively committed to develop for all students an understanding of American and Montana Indian people and their histories, as well as foster respect for their respective cultures.

Because of the unique position and place in American history, the American Indian peoples' role in the development of the United States, with emphasis on the experience of the Montana Tribes, shall be included wherever appropriate in the instruction of Missoula County Public School students, in accordance with the state Constitution and state standards. Instructions concerning the historic and current roles of Indian people shall be delivered in a respectful, informative, and sensitive manner. When the social studies curriculum and other curricula are updated according to the District's curriculum cycle, the written curriculum shall reflect this policy. Staff development will be provided pertinent to curriculum implementation.
NOTE: The District has nondiscriminatory policies in effect, which may be referenced.

Legal Reference:	Art. X, Sec. 1(2), Montana Constitution	
	§§ 20-1-501, et seq., MCA	Recognition of American Indian cultural heritage - legislative intent
10.55.603 ARM	Curriculum Development and Assessment	
10.55.701 ARM	Board of Trustees	
10.55.803 ARM	Learner Access	

Policy History:

History of Previous File 2121:	Presented to PN&P Committee for first reading, 3/30/00
	Approved First Reading, 4/11/00
	Presented to PN&P Committee for second reading, 4/27/00
	Revised at C&I Committee, 5/2/00
	Adopted on: October 10, 2000

Adopted on: January 14, 2003 (Policy recodified in Series 2000 adoption)

MONTANA OFFICE OF PUBLIC INSTRUCTION
INDIAN EDUCATION FOR ALL
HIGH SCHOOL LESSON PLANS

http://opi.mt.gov/Programs/IndianEd/curricsearch.html

Specific Grade Level	IEFA Math Lesson Title	URL Address
Grade 9	Pow Wow Trails	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%209%20Pow%20Wow%20Trails.pdf
Grade 9	Reservation Land Areas	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G%209%20Reservation%20Land%20Areas.pdf
Grade 10	Beading Patterns Using Reflections	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G10%20Beading%20Patterns%20Using%20Reflections.pdf
Grade 10	Tipi Geometry & Trigonometry	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G10%20Tipi%20Geometry%20&%20Trigonometry.pdf
Grade 11	Montana Native American Population	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G11%20Montana%20Native%20American%20Population.pdf
Grade 12	Seven Stars	http://www.opi.mt.gov/pdf/IndianEd/Search/Mathematics/G12%20Seven%20Stars.pdf

ALGEBRA 1 AND HONORS ALGEBRA 1
Grade 9

Unit of Credit: 1 Year (Required)

Prerequisite: None

Course Overview

Domains	**Seeing Structure in Expressions**	**Arithmetic with Polynomials and Rational Expressions**	**Creating Equations**	**Reasoning with Equations and Inequalities**
Clusters	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems	• Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	• Create equations that describe numbers or relationships	• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the imitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Expressions.

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05\,p$ shows that adding a tax is the same as multiplying the price by a constant factor. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities.
An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling.
Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Honors Algebra 1
Students successfully completing the Honors Algebra 1 course designation will cover the same standards below with greater depth. In addition, there are community service, career exploration, and research project components required.

Algebra 1 Enhancement
The Algebra 1 Enhancement course is designed to lend effective support to students concurrently enrolled in Algebra 1. Using the Response to Intervention (RtI) model, the Enhancement course is a Tier 2 Intervention aimed at students who are at-risk in mathematics. It allows for rapid response to student difficulties and provides opportunities for: additional time spent on daily targets, intensity of instruction, explicitly teaching and moving from the concrete to the abstract, frequent response from students and feedback from teachers, as well as strategic teaching using data to direct instruction. Students are placed in the Algebra 1 Enhancement course based on test scores, teacher/parent request, and academic achievement. These students are concurrently enrolled in Algebra 1. Students receive elective credit for the Algebra 1 Enhancement course.

Number and Quantity Content Standards

Domain: The Real Number System — N-RN

Cluster: Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
   - I can apply the properties of exponent to rational exponents.
   - I can explain how rational exponents follow from the properties of integer exponents. (See above)

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
   - I can write radical expressions using rational exponents and vice versa.

Cluster: Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
   - I can use the closure property or show by example the sum or product or two rational numbers are rational.
   - I can use the closure property or show by example the sum of a rational and an irrational number is irrational.
   - I can use the closure property or show by example the product or a nonzero rational number and an irrational is irrational.

Domain: Quantities — N-Q

Cluster: Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
   - I can interpret units in the context of the problem.
   - I can use unit analysis to check the reasonableness of my solution.

- I can choose and interpret an appropriate scale given data to be represented on a graph or display.

2. Define appropriate quantities for the purpose of descriptive modeling.
   - I can determine an appropriate quantity to model a situation.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
   - I can choose a level of accuracy appropriate to the measuring tool or situation.

Algebra Content Standards

Domain: Seeing Structure in Expressions **A-SSE**

Cluster: Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   - I can interpret expressions that represent a quantity in terms of its context.
   - I can identify the different parts of an expression and explain their meaning within the context of a problem.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
   - I can interpret expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*
   - I can rewrite algebraic expressions in equivalent forms such as factored or simplified form.
   - I can use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor an expression completely.
   - I can simplify expressions by combining like terms, using the distributive property and using other operations with polynomials.

Cluster: Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   - I can choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   - I can write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros.

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

- I can complete the square in a quadratic expression to convey the vertex form and determine the maximum or minimum value of the quadratic function, and to explain the meaning of the vertex.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

- I can use properties of exponents (such as power of a power, product of powers, power of a product, power of a quotient) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

Domain: Arithmetic with Polynomials and Rational Expressions A-APR

Cluster: Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
   - I can identify polynomials.
   - I can add, subtract, and multiply polynomials.
   - I can recognize how closure applies under these operations.

Domain: Creating Equations A-CED

Cluster: Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
   - I can create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
   - I can create equations in two or more variables to represent relationships between quantities.
   - I can graph equations in two variables on a coordinate plane and label the axes and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
   - I can write and use a system of equations and/or inequalities to solve a real world problem.
   - I can use equations and inequalities to represent problem constraints and objectives (linear programming).

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R*.*
   - I can solve multi-variable formulas or literal equations for a specific variable.

Domain: Reasoning with Equations and Inequalities **A-REI**

Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Cluster: Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Cluster: Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curse (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.⋆
    - I can calculate and justify the solution(s) to a system of equations using multiple methods.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions Content Standards

Domain: Interpreting Functions F-IF

Cluster: Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
   - I can use the definition of a function to determine whether a relationship is a function given a table, graph or words.
   - I can identify x as an element of the domain and $f(x)$ as an element in the range given the function f.
   - I can identify that the graph of the function f is the graph of the function $y=f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
   - I can use function notation, $f(x)$, when a relation is determined to be a function.
   - I can evaluate functions for inputs in their domains.
   - I can interpret statements that use function notation in terms of a context in which they are used.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
   - I can recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
   - I can write a recursive formula in function notation for a generated sequence.

Cluster: Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★
   - I can identify key features in graphs and tables to include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior for a linear, exponential and quadratic function.
   - I can sketch the graph of a function given its key features.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
   - I can interpret a graph to determine the appropriate numerical domain being described in the linear, exponential and quadratic functions.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.⋆

- I can calculate and interpret the average rate of change of a function presented symbolically or as a table.
- I can estimate the average rate of change over a specified interval of a function from its graph.

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- I can graph linear functions showing intercepts.
- I can graph quadratic functions showing intercepts, a maximum or a minimum.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- I can graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions.

e. Graph exponential showing intercepts.
- I can graph exponential functions, showing intercepts and end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- I can use the process of factoring and completing the square in a quadratic function to show zeros, a maximum or minimum, and symmetry of the graph, and interpret these in terms of a real-world situation.
- I can explain different properties of a function that are revealed by writing a function in equivalent forms.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
- I can use the properties of exponents to interpret exponential functions as growth or decay.
- I can identify the percent rate of change in an exponential function.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

- I can compare the key features of two linear, exponential, quadratic, absolute value, step and piecewise defined functions that are represented in different ways.

Domain: Building Functions F-BF

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   - I can write an explicit or recursive expression or describe the calculations needed to model a function given a situation.
   - I can write a linear, quadratic or exponential function that describes a relationship between two quantities.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   - I can combine function types, such as linear and exponential, using arithmetic operations.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.⋆
   - I can make connections between linear functions and arithmetic sequences, and exponential functions and geometric sequences.
   - I can write and translate between the recursive and explicit formula for a arithmetic sequence and use the formulas to model a situation.
   - I can write and translate between the recursive and explicit formula for a geometric sequence and use the formulas to model a situation

Cluster: Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
   - I can experiment to identify, using technology, the transformational effects on the graph of a function $f(x)$ (*linear, exponential, quadratic or absolute value functions*) when $f(x)$ is replaced by $f(x)+k$, $k\cdot f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k, both positive and negative.
   - I can find the value of k given the graph of a transformed function.
   - I can recognize even and odd functions from their graphs and equations.

4. Find inverse functions.

a. Solve an equation of the form $f(x) = ax + b$ for a simple function f that has an inverse and write an expression for the inverse. *For linear functions only.*
   - I can solve a linear function for the dependent variable and write the inverse of a linear
   - function by interchanging the dependent and independent variables.

<u>Domain: Linear, Quadratic, and Exponential Models F-LE</u>

Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

- I can determine a situation as linear or exponential by examining rates of change between data points.
- I can show there is a constant difference in a linear function over equal intervals.
- I can show there is a constant ratio in an exponential function over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- I can describe situations where one quantity grows or decays by a constant ratio per unit
- interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- I can describe situations where one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
    - I can write a linear or exponential function given an arithmetic or geometric sequence, a graph, a description of the relationship, or two points which can be read from a table.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
    - I can use graphs and tables to make the connection that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or any other polynomial function.

Cluster: Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.
    - I can explain the meaning of the coefficients, constants, factors, exponents, and intercepts in a linear or exponential function in terms of a context.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Statistics and Probability Content Standards

<u>Domain: Interpreting Categorical and Quantitative Data S-ID</u>

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
   - I can construct dot plots, histograms and box plots on a real number line.

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
   - I can describe a distribution using center and spread.
   - I can use the correct measure of center and spread to describe a distribution that is symmetric or skewed.
   - I can compare two or more different data sets using the center and spread of each.

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
   - I can identify outliers (extreme data points) using IQR and their effects on data sets.
   - I can interpret differences in different data sets in context.
   - I can interpret differences due to possible effects of outliers.

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
   - I can create a two-way table from two categorical variables and read values from a two-way table
   - I can interpret joint, marginal, and relative frequencies in context.
   - I can recognize associations and trends in data from a two-way table.

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear models. Discuss general principles referring to quadratic, and exponential models.
   - I can create a scatter plot from two quantitative variables.
   - I can describe the form (linear, quadratic or exponential), strength (strong to weak) and direction (positive or negative) of the relationship.
   - I can explain the meaning of slope and y-intercept (linear model) or the meaning of the growth rate and y-intercept (exponential model) or the meaning of the coefficients (quadratic model) in context.
   - I can use algebraic methods or technology to fit the data to a linear, exponential or quadratic function.

   b. Informally assess the fit of a function by plotting and analyzing residuals.
   - I can calculate a residual.

- I can create and analyze a residual plot.

c. Fit a linear function for a scatter plot that suggests a linear association.
  - I can use algebraic methods or technology to fit the data to a linear function.
  - I can use the function to predict values.

Cluster: Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
   - I can explain the meaning of the slope and y-intercept in context.

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
   - I can use a calculator or computer to find the correlation coefficient for a linear association.
   - I can interpret the meaning of the correlation coefficient in the context of the data.

9. Distinguish between correlation and causation.
   - I can explain the difference between correlation and causation.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Algebra 1 Montana Common Core Standards Vocabulary		
absolute value	factorization	quotient
acute	factor	radical
addition rule	frequency table	random sample
approximate	function	range
box plots	half-plane	rate of change
causation	histogram	ratio
center	horizontal	rational
closure	identity	rational expression
coefficient	independence	right triangle
communicative	inequality	root (zero)
complement	inference	sample space
conditional probability	intercept	scatter plot
constant	interpret	shape
constant rate	intersect	similar (triangle)
constraints	inverse	similarity
construct	irrational	slope
coordinate axis	linear equation	solution
coordinate plane	maxima	spread
correlation coefficient	minima	square
cube root	model	square root
curve	obtuse	subsets
data set	plot	systems of equality
derive	polynomial	systems of equation
domain	probability	table of values
dot plot	product	terms
equation	property	trigonometry
equivalent	Pythagorean Theorem	union
expression	quadratic equation	variable
extraneous	quantitative	vertical
function notation f(x)		

GEOMETRY AND HONORS GEOMETRY
Grades 9, 10

Unit of Credit: 1 Year

Prerequisite: Algebra 1

Course Overview:

Domains	**Congruence**	**Similarity, Right Triangles, and Trigonometry**	**Circles**	**Expressing Geometric Properties with Equations**	**Geometric Measurement and Dimension**	**Modeling with Geometry**
Clusters	• Experiment with transformations in the plane Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions	• Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply trigonometry to general triangles	• Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	• Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorems algebraically	• Explain volume formulas and use them to solve problems • Visualize relationships between two dimensional and three-dimensional objects	• Apply geometric concepts in modeling situations

Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The fundamental purpose of this Geometry course is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving toward formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school Montana Common Core Standards. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

The critical areas, organized into six units are as follows:

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

Critical Area 6: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Various cultural contexts such as American Indian designs in beadwork, star quilts, and tipis provide rich opportunities to apply concepts and skills of geometry.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations
The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Honors Geometry
Students successfully completing the Honors Geometry course designation will cover the same standards below with greater depth. (In addition, there are career exploration and research project components required.)

Geometry Enhancement
The Geometry Enhancement course is designed to lend effective support to students concurrently enrolled in Geometry. Using the Response to Intervention (RtI) model, the Enhancement course is a Tier 2 Intervention aimed at students who are at-risk in mathematics. It allows for rapid response to student difficulties and provides opportunities for: additional time spent on daily targets, intensity of instruction, explicitly teaching and moving from the concrete to the abstract, frequent response from students and feedback from teachers, as well as strategic teaching using data to direct instruction. Students are placed in the Enhancement course based on test scores, teacher/parent request, and academic achievement. These students are concurrently enrolled in Geometry. Students receive elective credit for the Geometry Enhancement course.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Geometry Content Standards

Domain: Congruence **G-CO**

Cluster: Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
    - I can use undefined terms (point, line, and plane) to precisely define angle, circle, perpendicular line, parallel line, and line segment.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
    - I can create transformations in the plane using a variety of tools and/or methods.
    - I can use rules to describe transformations as operations that map an image of the original figure.
    - I can compare isometries (transformations that preserve distance and angle, such as translations, reflections, and rotations) to non-isometries (such as dilations).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
    - I can describe the transformations that map a geometric figure onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
    - I can define rotations, reflections, and translations of geometric figures such as angles, circles, perpendicular lines, parallel lines, and line segments.
    - I can show that successive reflections over each of two intersecting lines result in a rotation and that successive reflections over each of two parallel lines result in a translation.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
    - I can draw the transformation (rotation, reflection, or translation) of a geometric figure using a variety of methods
    - I can create a sequence of transformations that map a given figure onto another.

Cluster: Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
    - I can use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion (translations and rotations) has on figures in the coordinate plane.
    - I can use the fact that rigid transformations preserve size and shape to connect the idea and definition of congruence.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
    - I can use the definition of congruence in terms of rigid motion (isometry) to show that two triangles are congruent if and only if all corresponding parts of the two triangles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
    - I can use the definition of congruence, based on rigid motion (isometry), to explain the criteria for triangle congruence (ASA, AAS, SSS, SAS, HL; does not include SSA).

Cluster: Prove geometric theorems.

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
    - I can prove theorems pertaining to lines and angles using two-column, flow, and/or paragraph proofs.

10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
    - I can prove theorems about triangles using two-column, flow, and/or paragraph proofs.

11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
    - I can prove theorems about parallelograms using two-column, flow, and/or paragraph proof.

Cluster: Make geometric constructions.

12. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including*

the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

- I can use a variety of tools and methods to perform the following constructions:
  - copy a line segment and an angle
  - bisect a line segment and an angle
  - construct perpendicular and parallel lines
  - construct a perpendicular bisector of a line segment
- I can identify ways geometric constructions were used by Montana American Indians.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
    - I can use a variety of tools and methods to construct the following:
      - an equilateral triangle
      - a square
      - a regular hexagon inscribed in a circle

<u>Domain: Similarity, Right Triangles, and Trigonometry G-SRT</u>

Cluster: Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   - I can verify experimentally the properties of dilations given by a center and a scale factor.
   - I can verify experimentally that, in a dilation, a line passing through the center of dilation is unchanged and a line that does not pass through the center of dilation is mapped onto a parallel line.

   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
   - I can verify experimentally that, when performing dilations of a line segment, the image segment becomes longer or shorter based on the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
   - I can use the definition of similarity transformations to determine whether two figures are similar.
   - I can explain similarity based on the equality of corresponding angles and the proportionality of corresponding sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
   - I can use the properties of similarity transformations to prove triangles are similar by AA criterion.

Cluster: Prove theorems involving similarity.

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
   - I can use AA, SAS, and SSS similarity theorems to prove triangles are similar.
   - I can use triangle similarity to prove other theorems about triangles, to include:
     - a line parallel to one side of a triangle divides the other two sides proportionally, and it's converse.
     - a line that divides two sides of a triangle proportionally, is parallel to the third side.
     - the Pythagorean Theorem using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
   - I can use similarity and congruence criteria to solve problems, including problems involving geometric figures.

Cluster: Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
   - I can show that the side ratios are the same in similar right triangles, which leads to the definition of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.
   - I can explain the relationship between the sine and cosine of complementary angles.
   - I can use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
   - I can use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster: Apply trigonometry to general triangles.

9. Derive the formula $A = ½\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
   - For a triangle that is not a right triangle, I can draw an auxiliary line from a vertex perpendicular to the opposite side and derive the formula for the area of a triangle ($A = ½\ ab\sin C$), using the fact that the height of the triangle is $h = a\sin C$.

10. Prove the Laws of Sines and Cosines and use them to solve problems.
    - I can prove the Laws of Sines and Cosines.
    - I can use the Laws of Sines and Cosines to solve problems.

11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
    - I can apply any method to find unknown measurements in right triangles.

- I can apply the Laws of Sines and Cosines to find unknown measurements in non-right triangles.

Domain: Circles G-C

Cluster: Understand and apply theorems about circles.

1. Prove that all circles are similar.
   - I can prove that all circles are similar. For example, I can use the fact that the ratio of diameter to circumference is the same for all circles.

2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
   - I can use definitions, properties, and theorems to:
     - identify and describe relationships among inscribed angles, radii, and chords, including central, inscribed, and circumscribed angles.
     - show that inscribed angles on a diameter are right angles.
     - show that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
   - I can construct the inscribed circle of a triangle.
   - I can construct the circumscribed circle of a triangle.
   - I can use definitions, properties, and theorems to prove properties of angles for a quadrilateral inscribed in a circle.

4. Construct a tangent line from a point outside a given circle to the circle.
   - I can construct a tangent line from a point outside a given circle to the circle.

Cluster: Find arc lengths and areas of sectors of circles.

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
   - I can use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius, identifying the constant of proportionality as the radian measure of the angle (arc length = $r\Theta$).
   - I can find the arc length of a circle.
   - I can use similarity to derive the formula for the area of a sector.
   - I can find the area of a sector.

Domain: Expressing Geometric Properties with Equations G-GPE

Cluster: Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- I can use the Pythagorean Theorem to derive the equation of a circle, given the center and radius.
- Given an equation of a circle, I can complete the square to find the center and radius.

2. Derive the equation of a parabola given a focus and directrix.
   - Given a focus and directrix, I can derive the equation of a parabola.
   - Given a parabola, I can identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is equidistant from the focus and the directrix.

Cluster: Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
   - I can use coordinates to prove geometric theorems algebraically.

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
   - I can prove that lines are parallel or perpendicular using slope.
   - I can solve geometric problems using the slope of parallel or perpendicular lines
   - I can find the equation of a line parallel or perpendicular to a given line that passes through a given point.

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
   - I can find the point on a directed line segment between two given points that partitions the segment in a given ratio.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.⋆
   - I can use coordinates and the distance formula to compute the perimeter of polygons and the areas of triangles and rectangles.

<u>Domain: Geometric Measurement and Dimension G-GMD</u>

Cluster: Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
   - I can explain the formulas for circumference of a circle and the area of circle by determining the meaning of each term. I can do this using dissection arguments, Cavalieri's principle, and informal limit arguments.
   - I can explain the formulas for volume of a cylinder, pyramid, and cone by determining the meaning of each term. I can do this using dissection arguments, Cavalieri's principle, and informal limit arguments.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.⋆
   - I can solve problems using volume formulas for cylinders, pyramids, cones, and spheres.

Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
   - I can identify two-dimensional cross-sections of three-dimensional objects.
   - I can identify three-dimensional objects generated by the rotation of two-dimensional objects.

Domain: Modeling with Geometry G-MG

Cluster: Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).⋆
   - I can use geometric shapes, their measures, and their properties to describe objects, such as modeling a Montana American Indian tipi as a cone.

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).⋆
   - I can apply the concept of density when referring to situations involving area and volume models, such as persons per square mile or BTU's per cubic foot.

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).⋆
   - I can solve problems by designing an object or structure that satisfies certain constraints, such as minimizing cost or the enlargement of a picture using grids, ratios, and proportions.

Statistics and Probability Content Standards

Domain: Conditional Probability and the Rules of Probability S-CP

Cluster: Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
   - I can define a sample space and events within the sample space.
   - I can identify subsets of a sample space given defined events, including unions, intersections, and complements of events.

2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
   - I can characterize two events, *A* and *B*, as independent if the probability of *A* and *B* occurring together is the product of their probabilities; that is, $P(A \text{ and } B) = P(A)P(B)$.

3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
   - I can show that the conditional probability of *A* given *B* is *P(A* and *B)/P(B).*
   - I can show that *A* and *B* are independent when the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

4. Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
   - I can construct and interpret two-way frequency tables of data for two categorical variables.
   - I can calculate probabilities from the table and use probabilities from the table to evaluate independence of two variables.

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
   - I can recognize and explain the concepts of independence and conditional probability in everyday situations.

Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B's* outcomes that also belong to *A*, and interpret the answer in terms of the model.
   - I can calculate conditional probabilities using the definition "the conditional probability of *A* given *B* is the fraction of *B*'s outcomes that also belong to *A*."
   - I can interpret conditional probability in context.

7. Apply the Addition Rule, P(A or B) = P(A)+ P(B) - P(A and B), and interpret the answer in terms of the model.
   - I can identify two events as disjoint (mutually exclusive).
   - I can calculate probabilities using the Addition Rule.
   - I can interpret probability in context.

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
   - I can calculate and interpret probabilities in context using the general Multiplication Rule.

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

- I can decide whether a permutation or combination is appropriate for calculating probabilities.
- I can use permutations and combinations in conjunction with other probability methods to calculate probabilities of compound events and solve problems.

Domain: Using Probability to Make Decisions S-MD

Cluster: Use probability to evaluate outcomes of decisions.

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
    - I can use expected values to help make good decisions.
    - I can use expected values to compare long-term benefits of several situations.

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
    - I can use probability concepts to analyze decisions made and strategies used in real-life situations.

Standards	Explanations and Examples
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Geometry Montana Common Core Standards Vocabulary			
AA similarity	corresponding sides	Law of Sines	rigid motion
alternate interior angle	cosine	line	rigid transformation
angle	cross section	line segment	rotation
arc length	cylinder	median·	SAS
area	density	midpoint	scale factor
ASA	diagonal	parabola	sector
auxiliary line	dilation	parallel lines	similar
axis of symmetry	directed line segment	parallelogram	sine
base angle	directrix	perimeter	slope
bisect	distance	perpendicular bisector	square
center	distance formula	perpendicular lines	SSS
central angle	endpoint	point	tangent
chord	equidistant	polygon	tangent line
circle	equilateral	pre-image	transformation
circumference	focus	proportion	translations
circumscribed angle	glide reflection	pyramid	transversal
circumscribed circle	hexagon	Pythagorean Theorem	trigonometry
complementary angle	HL	quadrilateral	vertex
complete the square	image	radius	vertical angle
cone	inscribed	rectangle	volume
constraint	inscribed circle	reflection	
coordinate geometry	isosceles triangle	regular	
corresponding angle	Law of Cosines	rhombus	

ALGEBRA 2 AND HONORS ALGEBRA 2
Grades 9, 10, 11, 12

Unit of Credit: 1 Year

Prerequisite: Geometry

Course Overview:

Domains	**Seeing Structure in Expressions**	**Arithmetic with Polynomials and Rational Expressions**	**Creating Equations**	**Reasoning with Equations and Inequalities**
Clusters	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems	• Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	• Create equations that describe numbers or relationships	• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

The critical areas for this course, organized into four units, are as follows:

Critical Area 1: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property.

Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area 3: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area 4: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Common Core State Standards for Mathematics Appendix A, page 37.

Expressions.

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities
An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$ are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling
Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Honors Algebra 2
Students successfully completing the Honors Algebra 2 course designation will cover the same standards below with greater depth. In addition, there are community service, career exploration, and research project components required.

Algebra 2 Enhancement
The Algebra 2 Enhancement course is designed to lend effective support to students concurrently enrolled in Algebra 2. Using the Response to Intervention (RtI) model, the Enhancement course is a Tier 2 Intervention aimed at students who are at-risk in mathematics. It allows for rapid response to student difficulties and provides opportunities for: additional time spent on daily targets, intensity of instruction, explicitly teaching and moving from the concrete to the abstract, frequent response from students and feedback from teachers, as well as strategic teaching using data to direct instruction. Students are placed in the Enhancement course based on test scores, teacher/parent request, and academic achievement. These students are enrolled in Algebra 2. Students receive elective credit for the Enhancement course.

Number and Quantity Content Standards

<u>Domain: The Complex Number System N-CN</u>

Cluster: Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
   - I can define and apply the properties of the imaginary number *i*.
   - I can write a complex number in the form of $a + bi$ where a and b are real numbers.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
   - I can add, subtract and multiply expressions involving complex numbers.
   - I can apply the commutative, associative and distributive properties to simplify expressions involving complex numbers.

Cluster: Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
   - I can solve quadratic equations that have complex solutions.

8. Extend polynomial identities to the complex numbers.
   - I can apply polynomial identities such as factoring to simplify expressions involving complex numbers.

9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
   - I can apply the Fundamental Theorem of Algebra to polynomial functions.

Algebra Content Standards

<u>Domain: Seeing Structure in Expressions A-SSE</u>

Cluster: Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   - I can distinguish parts of an expression, such as terms, factors, and coefficients.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
   - I can analyze complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it.
   - I can use the structure of an expression to reconstruct it in different forms.

Cluster: Write expressions in equivalent forms to solve problems.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
   - I can write a formula for the sum of a finite geometric series.
   - I can solve problems using a formula for the sum of a finite geometric series.

Domain: Arithmetic with Polynomials and Rational Expressions A-APR

Cluster: Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
   - I can demonstrate that polynomials are closed under the operations of integers.

Cluster: Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
   - I can divide polynomials and apply the Remainder Theorem using multiple strategies.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
   - I can identify zeros of a polynomial function and use them to graph the polynomial function.

Cluster: Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships.
   - I can apply and prove polynomial identities and use them to describe numerical relationships.

5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
   - I can apply the Binomial Theorem.

Cluster: Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
   - I can simplify and rewrite rational expressions in different forms.

7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
   - I can simplify rational expressions using addition, subtraction, multiplication, and division by nonzero rational expressions.

Domain: Creating Equations A-CED

Cluster: Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians..
   - I can compose and construct equations from a variety of contexts.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
   - I can create equations in two or more variables to represent relationships between quantities.
   - I can graph equations in two or more variables on the coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
   - I can write algebraic expressions and/or equations to represent constraints.
   - I can interpret solutions as viable or non-viable for a problem situation.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
   - I can solve an equation or formula for a variable.

Domain: Reasoning with Equations and Inequalities A-REI

Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
   - I can solve simple rational and radical equations in one variable.
   - I can distinguish between an actual solution and an extraneous solution.

Cluster: Represent and solve equations and inequalities graphically.

11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
    - I can calculate and justify the solution(s) to a system of equations using multiple methods.

Domain: Interpreting Functions F-IF

Cluster: Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
   - I can sketch a function that models a relationship between two quantities given a verbal description of the relationship.
   - I can interpret key features of the graph or table of a function.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
   - I can state the domain of a given function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

a. I can calculate and interpret the average rate of change of a function from a graph or table.

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   - I can graph square root, cube root and piecewise-defined functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and describe end behavior.
   - I can graph polynomial functions, identify zeros and describe end behavior.

   e. Graph exponential and logarithmic functions, show intercepts and describe end behavior, and graph trigonometric functions, describing period, mid-line, and amplitude.
   - I can graph exponential and logarithmic functions and label intercepts and describe end behavior.
   - I can graph trigonometric functions and describe and label period, mid-line, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   - I can factor a quadratic function to find zeros.
   - I can complete the square to find the zeros of a quadratic function.
   - I can relate the zeros of a quadratic function to identify extreme values, symmetry of the graph and interpret those in terms of a context.

   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
   - I can apply the properties of exponents to interpret expressions for exponential functions.
   - I can classify exponential expressions as exponential growth or decay.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)..
   - I can compare the properties of two functions where each function is represented in a different way such as algebraically, graphically, numerically in tables or by verbal descriptions.

Domain: Building Functions F-BF

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   b. Combine standard function types using arithmetic operations.
   - I can create an equation for a situation by combining multiple functions.

Cluster: Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
   - I can identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).
   - I can identify the value of k given the graphs of the function.
   - I can recognize even and odd functions from their graphs and create algebraic expressions for them.

4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
   - I can determine if a function has an inverse.
   - I can write an expression or equation for the inverse of a function.

<u>Domain: Linear, Quadratic, and Exponential Models F-LE</u>

Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, use a logarithm to solve $A=cb^{kt}$
   - I can evaluate exponential models using logarithms to re-write the equation or use technology to calculate the logarithm.

<u>Domain: Trigonometric Functions F-TF</u>

Cluster: Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
   - I can convert angles from radian measure to degrees and back on the unit circle.
   - I can identify and label radian measures on the unit circle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
   - I can use the coordinate plane to represent situations involving trigonometry.

Cluster: Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g. science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.
   - I can create a trigonometric function to model periodic phenomena from a variety of contexts with specified amplitude, frequency, and midline.

Cluster: Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.
   - I can prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Statistics and Probability

Domain: Interpreting Categorical and Quantitative Data S-ID

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and Montana American Indian data sources to estimate areas under the normal curve.
   - I can calculate the mean and standard deviation of a data set to fit it to a normal distribution and estimate population percentages.
   - I can differentiate between data sets that fit in a normal distribution and data sets that do not.
   - I can estimate the areas under the normal curve.

Domain: Making Inferences and Justifying Conclusions S-IC

Cluster: Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
   - I can predict population parameters based on a random sample from that population using statistics.

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
   - I can assess if a specified model is consistent using simulation.

Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
   - I can recognize the purposes of and differences among sample surveys, experiments and observational studies.

- I can explain how randomization relates to sample surveys, experiments and observational studies.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
   - I can estimate a population mean or proportion using data from a sample survey.
   - I can develop a margin of error for an estimated population mean or proportion through the use of simulation models for random sampling.

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
   - I can compare two treatments using data from a randomized experiment.
   - I can conclude if differences between parameters are significant by using simulations.

6. Evaluate reports based on data.
   - I can evaluate reports based on data.

Domain: Using Probability to Make Decisions S-MD

Cluster: Use probability to evaluate outcomes of decisions.

6. Use probabilities to make fair decisions.

7. Analyze decisions and strategies using probability concepts.

Standards	**Explanations and Examples**
Students are expected to:	**The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.**
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that

	take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Algebra 2 Montana Common Core Standards Vocabulary		
absolute value	function f(x)	quadratic formula
addition rule	geometric sequence	qualitative
algebraically	graphically	quantitative
amplitude	histogram	radian
annual rate	identities	radical
approximate	increasing (graph)	radius
arc	independent	range
arithmetic sequence	inequality	rate of decay
associative	infinite	rate of growth
box plot	integer	rational
causation	interest rate	rational expression
center	interpret	real number
center (data)	interquartile range	relative frequency
circle	intersection	recursive
coefficient	interval	relative minimum
commutative	inverse	remainder theorem
complement	irrational	roots
complete the square	linear	sample space
complex number	linear association	scatter plot
conditional probability	logarithmic	sequence
coordinate plane	maximum	series
correlation	mean	set
counterclockwise	median	shape (data)
decreasing (graph)	midline	spread (data)
degree	minimum	standard deviation
dependent	mode	statistic
distributive	normal distribution	subset
domain	odd function	subtended
dot plot	outliers	successive approximation
end behavior	parameter	symmetry
equations	period (graph)	table value
evaluate	polynomial	trigonometry
even function	probability	unit circle
exponential	proportion	zero (root)
expression	Pythagorean identity	
extreme value	Pythagorean theorem	
finite	Pythagorean triple	
frequency	quadratic	

DISCRETE MATH (MAT 115)
Grades 11, 12

Unit of Credit: 1 Semester

Prerequisite: Algebra 2 with a grade of C or higher

Course Overview:
The first half of this Discrete Math course includes topics describing linear functions and their applications, including solving systems of linear equations and linear programming. The second half of the course is an introduction to probability. Probability provides important foundations for the study of statistics and can help in the decision making process. This course covers the content of the Montana University System course, MAT 115 "Probability and Linear Mathematics" and includes an option for dual high school credit and UM-Missoula College credit.

Upon completion of the course, students will be able to perform each of the following:

- Master basic concepts of lines, linear systems, matrices, and linear programming (graphical method only);
- Understand basic probability concepts such as Venn diagrams, sample spaces with equally likely outcomes (counting principles), conditional probability (tree diagrams), Bayes' Theorem, binomial probabilities, and probability distributions;
- Understand the rudiments of statistics; measures of center and spread and the normal distribution;
- Use the above concepts to solve application problems.

Standard: Sets and Probability

a. I can use set builder notation to describe sets and subsets
b. I can describe sets using Venn diagrams, including union and intersection.
c. I can apply the Basic Probability Principle.
d. I can apply the Union Rule and Complement Rule for finding probability.
e. I can find Conditional Probability and the probability of Independent events.
f. I understand and can apply Bayes' Theorem to find probability.

Standard: Counting Principles

a. I can apply the multiplication principle to find the number of possible choices.
b. I can calculate the number of permutations of n-items.
c. I can calculate the number of combinations of n-items.
d. I can apply the counting principles to find probability.
e. I can calculate binomial probability.
f. I can calculate expected values using a probability distribution.

Standard: Statistics

a. I can construct a frequency distribution and histogram.
b. I can calculate the Measures of Central Tendency.
c. I can calculate the Measures of Variation.
d. I can apply normal distributions to find probability.

Standard: Linear Functions

a. I can find the equation for a line given two points.
b. I can find a linear function for business and economic applications.
c. I can find the Least Squares Line and the correlation coefficient.

Standard: Systems of Linear Equations and Matrices

a. I can perform operations on matrices (addition, subtraction, multiplication).
b. I can find the inverse of a matrix and use it to solve a system.

Standard: The Graphical Method of Linear Programming

a. I can graph linear inequalities and find the feasible region.
b. I can solve linear programming problems by developing and using the constraints.

FUNCTIONS, STATISTICS, AND TRIGONOMETRY (FST)
Grades 11, 12

Unit of Credit: 1 Year

Prerequisite: Algebra 2 or Consent of Instructor

Course Overview:
Fourth year mathematics courses offered at MCPS include Functions, Statistics, and Trigonometry (FST); Discrete Math; Pre-Calculus; AP Statistics; and AP Calculus.

The "college and career ready" line has been based on evidence from a number of sources, including international benchmarking, surveys of postsecondary faculty and employers, review of state standards, and expert opinion. Students meeting these standards through the first three years of high school mathematics should be well-prepared for introductory mathematics courses in 2- and 4- year colleges. Still, there are persuasive reasons for students to continue on to take a fourth mathematics course in high school.

Research consistently finds that taking mathematics above the Algebra 2 level highly corresponds to many measures of student success. In his groundbreaking report Answers in the Toolbox, Clifford Adelman found that the strongest predictor of postsecondary success is the highest level of mathematics completed (Executive Summary). ACT has found that taking more mathematics courses correlates with greater success on their college entrance examination. Of students taking (Algebra 1, Geometry, Algebra 2, and no other high school mathematics courses), only thirteen percent met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking Calculus met the benchmark (ACTb 13).

High school students should be encouraged to select from a range of high quality mathematics options. STEM (Science, Technology, Engineer, Mathematics)-intending students should be strongly encouraged to take Pre-Calculus and Calculus (and perhaps a computer science course). A student interested in psychology may benefit greatly by taking FST followed by AP Statistics. A student interested in starting a business after high school could use knowledge and skills gleaned from a course on mathematical decision-making. Mathematically-inclined students can, at this level, double up on courses—a student taking college calculus and college statistics would be well-prepared for almost any post-secondary career.

Taken together, there is compelling rationale for urging students to continue their mathematical education throughout high school, allowing students several rich options once they have demonstrated mastery of core content.

Standards marked with a (+) may appear either in courses required for all students, or in later courses. In particular, the (+) standards can form the starting point for fourth year courses.

Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards.

Number and Quantity Content Standards

Domain: The Complex Number System N-CN

Cluster: Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
   - I can write the conjugate of a complex number.
   - I can use conjugates to find moduli and quotients of complex numbers.

Cluster: Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
   - I can represent complex numbers on the complex plane in rectangular and polar form (including real and an represent a complex number and its conjugate on the complex plane.
   - I can raise a complex number to an integer power.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
   - I can calculate the distance between numbers in the complex plane as the modulus of the difference.
   - I can calculate the midpoint of a segment in the complex plane.

Domain: Vector Quantities and Matrices N-VM

Cluster: Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
   - I can identify the magnitude and direction of a vector.
   - I can use the appropriate symbols.
   - I can represent a vector in the coordinate plane.

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
   - I can calculate the components (horizontal and vertical) using the initial point and the terminal point.

3. (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.
   a. I can solve problems where quantities are represented by vectors.

Cluster: Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   - I can add vectors end-to-end, component-wise, and by the parallelogram rule.

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   - I can calculate the magnitude and direction of the sum of two vectors.

   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
   - I can calculate vector subtraction using the additive inverse.
   - I can graphically represent vector subtraction.

5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   - I can graphically represent scalar multiplication of vectors.

   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
   - I can calculate the magnitude of a scalar multiple.

Cluster: Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
   - I can represent data using matrices.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
   - I can multiply a matrix by a scalar.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
   - I can determine if two matrices can be added or subtracted (same dimensions).
   - I can determine if two matrices can be multiplied (same inner dimensions).
   - I can add/subtract/multiply two matrices.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
   - I can demonstrate that multiplication of matrices is not commutative.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
    - I can add the zero matrix or identity matrix.

- I can calculate the determinant of a square matrix and if it is non-zero, then that matrix has an inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
    - I can multiply a vector (with one column) by a matrix.

12. (+) Work with 2 × 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
    - I can determine the area of a parallelogram by taking the absolute value of the matrix formed by the component vectors of two consecutive vertices.

Algebra Content Standards

Domain: Reasoning with Equations and Inequalities A-REI

Cluster: Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
    - I can write a matrix equation to represent a system of linear equations.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
    - I can use matrices to solve a system of linear equations.

Functions Content Standards

Domain: Interpreting Functions F-IF

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    - I can graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Domain: Building Functions F-BF

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆
    c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
    - I can apply composite functions to real life situations.

Cluster: Build new functions from existing functions.

4. Find inverse functions.
    b. (+) Verify by composition that one function is the inverse of another.
    - I can verify that a composition of one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

- I can determine the inverse values from a graph or a table.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

- I can determine the inverse of a function that is not one-to-one by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
    - I can solve problems using logarithms and exponents.

Domain: Trigonometric Functions F-TF

Cluster: Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number.
    - I can use special triangles to construct the unit circle.
    - I can use the unit circle to express the values of sine, cosines, and tangent for *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
    - I can use the unit circle to identify symmetry (odd and even) and periodicity of trigonometric functions.

Cluster: Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
    - I can graph the inverse of a trigonometric function.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.⋆
    - I can use trigonometric functions to model real life situations.
    - I can use inverse trigonometric functions to solve equations.

Cluster: Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
    - I can prove the addition and subtraction formulas for sine, cosine, and tangent.
    - I can use the addition and subtraction formulas for sine, cosine, and tangent to solve problems.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Geometry Content Standards

Domain: Expressing Geometric Properties with Equations G-GPE

Cluster: Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices.
   - I can determine the equation of an ellipse given the foci and the directrices.
   - I can determine the equation of a hyperbola given the foci and directrices.

Cluster: Explain volume formulas and use them to solve problems.

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
   - I can use Cavalieri's principle to derive the volume formula for a sphere.

Statistics and Probability Content Standards

Domain: Using Probability to Make Decisions S-MD

Cluster: Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
   - I can define a random variable.
   - I can construct a probability distribution table.
   - I can graph a probability distribution table.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
   - I can calculate and interpret in context the expected value of a discrete random variable.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
   - I can develop a theoretical probability distribution and calculate the expected value.

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
   - I can develop an empirical probability distribution and calculate the expected value.

Cluster: Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   - I can set up a probability distribution for a random variable representing the pay-off values in a game of chance.

   a. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
   - I can compare strategies using expected values with respect to decision making.

PRE-CALCULUS AND HONORS PRE-CALCULUS
Grades 9, 10, 11, 12

Unit of Credit: 1 Year

Pre-requisite: Algebra 2

Course Overview:
Pre-Calculus will review and tie together all aspects of previous high school math courses. It will also prepare students for success in Calculus.

The "college and career ready" line has been based on evidence from a number of sources, including international benchmarking, surveys of post-secondary faculty and employers, review of state standards, and expert opinion. Students meeting these standards through the first three years of high school mathematics should be well-prepared for introductory mathematics courses in 2- and 4- year colleges. Still, there are persuasive reasons for students to continue on to take a fourth mathematics course in high school.

Research consistently finds that taking mathematics above the Algebra 2 level highly corresponds to many measures of student success. In his groundbreaking report Answers in the Toolbox, Clifford Adelman found that the strongest predictor of post-secondary success is the highest level of mathematics completed (Executive Summary). ACT has found that taking more mathematics courses correlates with greater success on their college entrance examination. Of students taking (Algebra 1, Geometry, Algebra 2, and no other mathematics courses), only thirteen percent of met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking Calculus met the benchmark (ACTb 13).

High school students should be encouraged to select from a range of high quality mathematics options. STEM (Science, Technology, Engineering, Mathematics)-intending students should be strongly encouraged to take Pre-Calculus and Calculus (and perhaps a computer science course). A student interested in psychology may benefit greatly by taking FST followed by AP Statistics. A student interested in starting a business after high school could use knowledge and skills gleaned from a course on mathematical decision-making. Mathematically-inclined students can, at this level, double up on courses—a student taking college calculus and college statistics would be well-prepared for almost any post-secondary career.

Taken together, there is compelling rationale for urging students to continue their mathematical education throughout high school, allowing students several rich options once they have demonstrated mastery of core content.

Standards marked with a (+) may appear either in courses required for all students, or in later courses. In particular, the (+) standards can form the starting point for fourth year courses.

Number and Quantity Content Standards

<u>Domain: The Complex Number System N-CN</u>

Cluster: Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
   - I can write the conjugate of a complex number.
   - I can use conjugates to find moduli and quotients of complex numbers.

Cluster: Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
   - I can represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).
   - I can explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.*
   - I can add, subtract, multiply, and divide complex numbers.
   - I can represent a complex number and it's conjugate on the complex plane.
   - I can raise a complex number to an integer power.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
   - I can calculate the distance between numbers in the complex plane as the modulus of the difference.
   - I can calculate the midpoint of a segment in the complex plane.

<u>Domain: Vector Quantities and Matrices N-VM</u>

Cluster: Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$, v).
   - I can identify the magnitude and direction of a vector.
   - I can use the appropriate symbols.
   - I can represent a vector in the coordinate plane.

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
   - I can calculate the components (horizontal and vertical) using the initial point and the terminal point.

3. (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.
   - I can solve problems where quantities are represented by vectors.

Cluster: Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   - I can add vectors end-to-end, component-wise, and by the parallelogram rule.

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   - I can calculate the magnitude and direction of the sum of two vectors.

   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
   - I can calculate vector subtraction using the additive inverse.
   - I can graphically represent vector subtraction.

5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   - I can graphically represent scalar multiplication of vectors.

   b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
   - I can calculate the magnitude of a scalar multiple.

Cluster: Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
   - I can represent data using matrices.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
   - I can multiply a matrix by a scalar.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
   - I can determine if two matrices can be added or subtracted (same dimensions).
   - I can determine if two matrices can be multiplied (same inner dimensions).
   - I can add/subtract/multiply two matrices.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
   - I can demonstrate that multiplication of matrices is not commutative.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
   - I can add the zero matrix or identity matrix.
   - I can calculate the determinant of a square matrix and if it is non-zero, then that matrix has an inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
   - I can multiply a vector (with one column) by a matrix.

12. (+) Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
   - I can determine the area of a parallelogram by taking the absolute value of the matrix formed by the component vectors of two consecutive vertices.

Algebra Content Standards

Domain: Reasoning with Equations and Inequalities A-REI

Cluster: Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
   - I can write a matrix equation to represent a system of linear equations.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).
   - I can use matrices to solve a system of linear equations.

Functions Content Standards

Domain: Interpreting Functions F-IF

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   - I can graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   - I can locate the roots of polynomial equations using a variety of methods including synthetic division, factoring and graphing.
   - I can sketch the curves of polynomial functions without the aide of technology.

Domain: Building Functions **F-BF**

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆
   - c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
   - I can apply composite functions to real life situations.

Cluster: Build new functions from existing functions.

4. Find inverse functions.
   - b. (+) Verify by composition that one function is the inverse of another.
   - I can verify that a composition of one function is the inverse of another.
   - c.(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   - I can determine the inverse values from a graph or a table.
   - d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
   - I can determine the inverse of a function that is not one-to-one by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
   - I can solve problems using logarithms and exponents.
   - I can model and graph data using exponential and logarithmic functions.

Domain: Trigonometric Functions **F-TF**

Cluster: Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
   - I can use special triangles to construct the unit circle.
   - I can use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
   - I can use the unit circle to identify symmetry (odd and even) and periodicity of trigonometric functions.

Cluster: Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
   - I can graph the inverse of a trigonometric function.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.⋆
   - I can use trigonometric functions to model real life situations.
   - I can use inverse trigonometric functions to solve equations.

Cluster: Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

- I can prove the addition and subtraction formulas for sine, cosine, and tangent.
- I can use the addition and subtraction formulas for sine, cosine, and tangent to solve problems.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Geometry Content Standards

Domain: Expressing Geometric Properties with Equations G-GPE

Cluster: Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices.

- I can determine the equation of an ellipse given the foci and the directrices.
- I can determine the equation of a hyperbola given the foci and directrices.

Cluster: Geometric Measurement and Dimension.

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

- I can use Cavalieri's principle to derive the volume formula for a sphere.

Statistics and Probability Content Standards

Domain: Using Probability to Make Decisions S-MD

Cluster: Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

- I can define a random variable.
- I can construct a probability distribution table.
- I can graph a probability distribution table.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
   - I can calculate and interpret in context the expected value of a discrete random variable.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
   - I can develop a theoretical probability distribution and calculate the expected value.

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
   - I can develop an empirical probability distribution and calculate the expected value.

Cluster: Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   - I can set up a probability distribution for a random variable representing the pay-off values in a game of chance.

   a. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
   - I can compare strategies using expected values with respect to decision making.

Standards	**Pre-Calculus Montana Common Core Standards Vocabulary**
9-12.CN.3	conjugate, complex number, quotient, denominator, modulus
9-12.CN.4	complex number, complex plane, rectangular coordinates, polar coordinates, coordinate plane, modulus, argument
9-12.CN.5	complex number, complex plane, rectangular coordinates, conjugation, modulus, argument
9-12.CN.6	complex number, complex plane, difference, modulus, distance, midpoint, line segment, average, real, imaginary, endpoint
9-12.CN.7	quadratic equation, factor, complete the square, quadratic formula, real number coefficient, complex solution, complex roots, discriminant
9-12.CN.8	sum of squares, factor, real number, complex number, polynomial

9-12.CN.9	Fundamental Theorem of Algebra, Linear Factorization Theorem, quadratic, polynomial, linear, factor, complex number, zeros
9-12.VM.1	vector, magnitude, direction, directed line segment, initial point, terminal point
9-12.VM.2	component form, vector, initial point, terminal point, formula
9-12.VM.3	vector, component form, magnitude, direction, magnitude and direction form
9-12.VM.4	vector, component form, coordinate plane, magnitude, direction, Parallelogram Rule, initial point, terminal point, sum, magnitude and direction form, additive inverse, triangle inequality
9-12.VM.5	vector, component form, coordinate plane, magnitude, direction, scalar multiple, absolute value
9-12.VM.6	matrix, dimensions of a matrix, row, column
9-12.VM.7	matrix, dimensions of a matrix, row, column, scalar multiplication, scalar
9-12.VM.8	matrix, dimensions of a matrix, row, column
9-12.VM.9	commutative property of multiplication, associative property of multiplication, distributive property, matrix, square matrix, dimensions of a matrix, row, column
9-12.VM.10	matrix, square matrix, dimensions of a matrix, row, column, identity property of addition, identity property of multiplication, zero matrix, identity matrix, determinant, inverse
9-12.VM.11	vector, matrix, dimensions of a matrix, row, column, transform, translate, dilation, reflect, rotate
9-12.GPE.3	ellipse, hyperbola, focus, distance formula, sum, length, axes (major and minor), difference
9-12.GMD.2	height, base area, volume, Cavalieri’s Principle, cross sectional area
9-12.MD.1	data, random variable, sample space, outcome, probability, probability distribution, event, histogram, independent variable, dependent variable
9-12.MD.2	expected value, mean, random variable, probability distribution
9-12.MD.3	probability distribution, random variable, sample space, theoretical probability, expected value, outcome
9-12.MD.4	probability distribution, random variable, sample space, outcome, probability, expected value
9-12.MD.5	outcomes, probability expected value, expected payoff, probability distribution table

TECHNICAL MATHEMATICS (MAT 111)
Grades 11, 12

Unit of Credit: 1 Year

Pre-requisite: Geometry

Course Overview:
This course is designed for those interested in pursuing a technical or vocational career or degree after high school. The course includes fractions, decimals, ratios, proportions, formulas, and word problems. Topics studied are metric and American measurements systems, linear equations, algebra, developing applied skills in practical geometry, solid figures, and basic trigonometry. This course is offered with an option of Dual Credit through UM Missoula College: MAT 111 Technical Mathematics combining 1 high school math credit with 3 university math credits. Additional requirements and/or UM Missoula College fees may apply.

Number and Quantity Content Standards

Domain: The Real Number System **N-RN**

Cluster: Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Cluster: Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Domain: Quantities **N-Q**

Cluster: Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra Content Standards

Domain: Seeing Structure in Expressions **A-SSE**

Cluster: Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Cluster: Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Domain: Arithmetic with Polynomials and Rational Expressions **A-APR**

Cluster: Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Domain: Creating Equations **A-CED**

Cluster: Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Domain: Reasoning with Equations and Inequalities **A-REI**

Cluster: Understand solving equations as a process of reasoning and explain reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Cluster: Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x – p)2 = q$ that has the same solutions. Drive the quadratic formula from this form.

   b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Cluster: Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x2 + y2 = 3$.

Cluster: Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the *x*-coordinates of the points where the graphs of the equations $y – f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential.

Functions Content Standards

Domain: Building Functions **F-BF**

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Geometry Content Standards

Domain: Congruence G-CO

Cluster: Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Domain: Similarity, Right Triangles, and Trigonometry G-SRT

Cluster: Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster: Apply trigonometry to general triangles.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Domain: Geometric Measurement and Dimension G-GMD

Cluster: Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.⋆

Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Domain: Modeling with Geometry G-MG

Cluster: Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).⋆

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).⋆

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).⋆

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Statistics and Probability Content Standards

Domain: Interpreting Categorical and Quantitative Data S-ID

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

ADVANCED PLACEMENT (AP) STATISTICS
Grades 10, 11, 12

Unit of Credit: 1 Year

Pre-requisite: Algebra 2

Course Overview:
The purpose of this AP Statistics course is to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data.

Students are exposed to four broad conceptual themes:
- Exploring Data - Describing patterns and departures from patterns,
- Sampling and Experimentation - Planning and conducting a study,
- Anticipating Patterns - Exploring random phenomena using probability and simulation,
- Statistical Inference - Estimating population parameters and testing hypotheses.

Students who successfully complete the course and exam may receive credit, advanced placement, or both for a one-semester introductory college statistics course.

Exploring Data

Standard: Construct and interpret graphical displays of distributions of univariate data (i.e. dotplots, stemplots, histograms, and cumulative frequency plots).
- I can calculate and interpret measures of center and spread.
- I can identify clusters, gaps, outliers, and other unusual features of a data set from a graphical display.

Standard: Summarize distributions of univariate data.
- I can calculate and interpret measures of center (median, mean), spread (range, interquartile range, standard deviation) and position (quartiles, percentiles, z-scores) of a data set.
- I can construct and interpret boxplots.
- I can explain the effect of changing units on summary measures.

Standard: Compare distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots).
- I can compare and interpret center and spread, clusters and gaps, outliers, and shapes within groups of data and between data sets.

Standard: Explore bivariate data.
- I can interpret the correlation of a scatterplot and relate it to the linearity of the data.
- I can produce a least-squares regression line for a set of bivariate data.
- I can identify residual plots, outliers, and influential points of a data set.
- I can apply logarithmic and power transformations to a set of data to achieve linearity.

Standard: Explore categorical data.

- I can produce and interpret tables and bar charts.
- I can explain marginal and joint frequencies for two-way tables.
- I can interpret conditional relative frequencies and associations of categorical data.
- I can compare distributions using bar charts.

Sampling and Experimentation

Standard: Collect data according to a well-developed plan.

- I can understand methods of data collection. *Examples: census, sample survey, experiment, observational study*

Standard: Plan and conduct surveys.

- I can explain and justify the characteristics of a well-designed and well-conducted survey.
- I can identify populations, samples and random selection techniques for a survey.
- I can identify sources of bias in sampling and surveys.
- I can identify and evaluate sampling methods.
  Examples: simple random sampling, stratified random sampling, cluster sampling

Standard: Plan and conduct experiments.

- I can identify and evaluate the characteristics of a well-designed and well-conducted experiment. *Examples: treatments, control groups, experimental units, random assignments, replication*
- I can identify and discuss sources of bias and confounding, including placebo effect and blinding.
- I can identify and explain completely randomized design, randomized block design, and matched pairs design.

Standard: Generalize results and conclusions.

- I can generalize the results and discuss the types of conclusions that can be drawn from observational studies, experiments, and surveys.

Anticipating Patterns

Standard: Use probability to anticipate what the distribution of data should look like under a given model.

- I can interpret probability measures, including long-run relative frequencies and the Law of Large Numbers.
- I can apply the addition rule, multiplication rule, conditional probability and independence to calculate the probability of an event.
- I can understand discrete random variables and their probability distributions, including binomial and geometric distributions.
- I can simulate random behavior and probability distributions.

- I can calculate the expected value (mean) and standard deviation of a random variable and linearly transform a random variable.

Standard: Combine independent random variables.
- I can distinguish between independent and dependent events.
- I can find the mean and standard deviation for sums and differences of independent random variables.

Standard: Understand and apply the normal distribution.
- I can explain the properties of the normal distribution.
- I can use tables of the normal distribution to solve problems
- I can apply the normal distribution as a model for measurements.

Standard: Understand and apply sampling distributions.
- I can explain and apply the sampling distribution of a sample proportion and a sample mean.
- I understand and can apply the Central Limit Theorem.
- I can produce the sampling distribution of a difference between two independent sample proportions and that of a difference between two independent sample means.
- I can conduct and evaluate a simulation of sampling distributions.
- I can explain and apply a t-distribution and a chi-square distribution.

Statistical Inference

Standard: Estimate population parameters and test hypotheses.
- I can estimate population parameters and margins of error.
- I can discuss the properties of point estimators, including unbiasedness and variability.
- I can apply and explain the properties of confidence levels and confidence intervals.
- I can find and apply a large sample confidence interval for a proportion of for a difference between two proportions.
- I can find the confidence interval for a mean, a difference between two means (paired and unpaired) and for the slope of a least-squares regression line.

ADVANCED PLACEMENT (AP) CALCULUS AB
Grades 11, 12

Unit of Credit: 1 Year

Pre-requisite: Pre-Calculus

Course Overview:
This topic outline is intended to indicate the scope of the course, but is not necessarily the order in which the topics must be taught. Although the AP exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

Functions, Graphs, and Limits Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Standard: Limits of functions (including one-sided limits).

- I can intuitively understand the limiting process.
- I can calculate limits using algebra.
- I can estimate limits from graphs or tables of data.

Standard: Asymptotic and unbounded behavior.

- I can understand asymptotes in terms of graphical behavior.
- I can describe asymptotic behavior in terms of limits involving infinity.
- I can compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Standard: Continuity as a property of functions.

- I can intuitively understand the definition of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- I can understand continuity in terms of limits.
- I can understand geometric graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

Derivatives

Standard: Concept of the derivative.

- I can represent graphically, numerically, and analytically the derivative.
- I can interpret the derivative as an instantaneous rate of change.
- I can define the derivative as the limit of the difference quotient.
- I can give examples of how differentiability and continuity are related.

Standard: Derivative at a point.

- I can calculate the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- I can write an equation of the tangent line to a curve at a point and use the equation to calculate a linear approximation.
- I can calculate the instantaneous rate of change as the limit of the average rate of change.
- I can estimate the approximate rate of change from graphs and tables of values.

Standard: Derivative as a function.

- I can describe the distinguishing characteristics of the graphs of f and f' at corresponding x-values.
- I can describe the relationship between the increasing and decreasing behavior of f and the sign of f'.
- I can state and apply the Mean Value Theorem and its geometric interpretation.
- I can write equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Standard: Second derivatives.

- I can describe corresponding characteristics of the graphs of f, f', and f''.
- I can state the relationship between the concavity of f and the sign of f''.
- I can calculate` points of inflection as places where concavity changes.

Standard: Applications of derivatives.

- I can analyze curves, including the notions of monotonicity and concavity.
- I can optimize both absolute (global) and relative (local) extrema.
- I can model rates of change, including related rates problems.
- I can use implicit differentiation to find the derivative of an inverse function.
- I can interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- I can interpret geometric interpretation of differential equations via direction fields and the relationship between direction fields and solution curves for differential equations.

Standard: Computation of derivatives.

- I can calculate derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- I can apply derivative rules for sums, products, and quotients of functions.
- I can apply the chain rule and implicit differentiation techniques.

Integrals

Standard: Interpretations and properties of definite integrals.

- I can calculate the definite integral as a limit of Riemann sums.
- I can calculate a definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
  $\int_a^b f'(x)dx = f(b) - f(a)$.
- I can apply basic properties of definite integrals (examples include additivity and linearity).

Standard: Applications of integrals.

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid of revolution using the disk and washer method, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Standard: Fundamental Theorem of Calculus.

- I can use of the Fundamental Theorem to evaluate definite integrals.
- I can use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Standard: Techniques of antidifferentiation.

- I can calculate antiderivatives following directly from the derivatives of basic functions.
- I can evaluate antiderivatives by substitution of variables (including changing the limits for definite integrals).

Standard: Applications of antidifferentiation.

- I can find specific antiderivatives using initial conditions, including applications to motion along a line.
- I can solve separable differential equations and use them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Standard: Numerical approximations to definite integrals

- I can use Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

ADVANCED PLACEMENT (AP) CALCULUS BC
Grades 11, 12

Unit of Credit: 1 Year

Pre-requisite: Pre-Calculus

Course Overview:
The topic outline for Calculus BC includes all Calculus AB topics. Additional topics that are BC topics are found in paragraphs marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

Functions, Graphs, and Limits Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Standard: Limits of functions (including one-sided limits).
- I can intuitively understand the limiting process.
- I can calculate limits using algebra.
- I can estimate limits from graphs or tables of data.

Standard: Asymptotic and unbounded behavior.
- I can understand asymptotes in terms of graphical behavior.
- I can describe asymptotic behavior in terms of limits involving infinity.
- I can compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Standard: Continuity as a property of functions.
- I can intuitively understand the definition of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- I can understand continuity in terms of limits.
- I can understand geometric graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

*** Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

Derivatives

Standard: Concept of the derivative.

- I can represent graphically, numerically, and analytically the derivative.
- I can interpret the derivative as an instantaneous rate of change.
- I can define the derivative as the limit of the difference quotient.
- I can give examples of how differentiability and continuity are related.

Standard: Derivative at a point.

- I can calculate the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- I can write an equation of the tangent line to a curve at a point and use it to calculate a linear approximation.
- I can calculate the instantaneous rate of change as the limit of the average rate of change.
- I can estimate the approximate rate of change from graphs and tables of values.

Standard: Derivative as a function.

- I can describe distinguishing characteristics of the graphs of f and f' at corresponding x-values.
- I can describe the relationship between the increasing and decreasing behavior of f and the sign of f'.
- I can state and apply the Mean Value Theorem and its geometric interpretation.
- I can write equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Standard: Second derivatives.

- I can describe corresponding characteristics of the graphs of f, f', and f''.
- I can state the relationship between the concavity of f and the sign of f''.
- I can calculate points of inflection as places where concavity changes.

Standard: Applications of derivatives.

- I can analyze curves, including the notions of monotonicity and concavity.
- + I can analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.
- I can optimize, both absolute (global) and relative (local) extrema.
- I can model rates of change, including related rates problems.
- I can use implicit differentiation to find the derivative of an inverse function.
- I can interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- I can interpret geometric interpretation of differential equations via direction fields and the relationship between direction fields and solution curves for differential equations.
- + I can use numerical solutions of differential equations according to Euler's method.
- + I can apply L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.

Standard: Computation of derivatives.

- I can calculate derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- I can apply derivative rules for sums, products, and quotients of functions.
- I can apply the chain rule and implicit differentiation techniques.
- **+** I can calculate derivatives of parametric, polar, and vector functions.

Integrals

Standard: Interpretations and properties of definite integrals.

- I can calculate a definite integral as a limit of Riemann sums.
- I can calculate a definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

  $\int_a^b f'(x)dx = f(b) - f(a)$.
- I can apply basic properties of definite integrals (examples include additivity and linearity).

*** Applications of integrals.**

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid of revolution using the disk and washer methods and the shell method, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Standard: Fundamental Theorem of Calculus.

- I can use of the Fundamental Theorem to evaluate definite integrals.
- I can use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Standard: Techniques of antidifferentiation.

- I can evaluate antiderivatives following directly from the derivatives of basic functions.
- I can evaluate antiderivatives by substitution of variables (including changing the limits for definite integrals).
- +I can use the techniques of integration by parts and simple partial fractions (non-repeating linear factors only).
- +I can evaluate improper integrals (as limits of definite integrals).

Standard: Applications of antidifferentiation.

- I can find specific antiderivatives using initial conditions, including applications to motion along a line.
- I can solve separable differential equations and use them in modeling (including the study of the equation $y' = ky$ and exponential growth).
- + I can solve logistic differential equations and use them in modeling.

Standard: Numerical approximations to definite integrals.

- I can use Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

*Polynomial Approximations and Series

Standard: * Concept of series.

A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

Standard: * Series of constants.

- + I can motivate examples of series, including decimal expansion.
- + I can use geometric series with some applications.
- + I can use the harmonic series.
- + I can use the alternating series and apply the error bound.
- + I can use terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of a *p*-series.
- + I can use the ratio test for convergence and divergence.
- + I can use the comparison test for series to test for convergence or divergence.

Standard: * Taylor series.

- + I can use Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).
- + I can use the Maclaurin series and the general Taylor series centered at $x = a$.
- + I can write the Maclaurin series for the functions e^x, $\sin(x)$, $\cos(x)$, and $1/(1-x)$.
- + I can use formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
- + I can use functions defined by power series.
- + I can calculate the radius and interval of convergence of a power series.
- + I can use the Lagrange error bound for Taylor polynomials.

INTERNATIONAL BACCALAUREATE (IB) MATHEMATICS COURSES

IB Learner Profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:	
Inquirers	They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.
Knowledgeable	They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.
Thinkers	They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.
Communicators	They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.
Principled	They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.
Open-minded	They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.
Caring	They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.
Risk-takers	They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.
Balanced	They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.
Reflective	They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Summary of Courses Available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence to understand better their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following factors:

- their own abilities in mathematics and the type of mathematics in which they can be successful
- their own interest in mathematics and those particular areas of the subject that may hold the most interest for them
- their other choices of subjects within the framework of the Diploma Programme
- their academic plans, in particular the subjects they wish to study in future
- their choice of career.

Teachers are expected to assist with the selection process and to offer advice to students.

Mathematical Studies Standard Level

This course is available only at standard level, and is equivalent in status to mathematics SL, but addresses different needs. It has an emphasis on applications of mathematics, and the largest section is on statistical techniques. It is designed for students with varied mathematical backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts. These students may need to utilize the statistics and logical reasoning that they have learned as part of the mathematical studies SL course in their future studies.

Mathematics Standard Level

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

IB Mathematics Courses Aims

The aims of all mathematics courses are to enable students to:

1. enjoy mathematics, and develop an appreciation of the elegance and power of mathematics

2. develop an understanding of the principles and nature of mathematics

3. communicate clearly and confidently in a variety of contexts

4. develop logical, critical and creative thinking, and patience and persistence in problem-solving

5. employ and refine their powers of abstraction and generalization

6. apply and transfer skills to alternative situations, to other areas of knowledge and to future developments

7. appreciate how developments in technology and mathematics have influenced each other

8. appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics

9. appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives

10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course.

Assessment Objectives

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematical studies SL course, students will be expected to demonstrate the following.

1. **Knowledge and understanding**: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.

2. **Problem-solving**: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.

3. **Communication and interpretation**: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.

4. **Technology**: use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.

5. **Reasoning**: construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.

6. **Investigative approaches**: investigate unfamiliar situations involving organizing and analyzing information or measurements, drawing conclusions, testing their validity, and considering their scope and limitations.

The Diploma Programme Hexagon

The course is presented as six academic areas enclosing a central core (see figure 1). It encourages the concurrent study of a broad range of academic areas. Students study: two modern languages (or a modern language and a classical language); a humanities or social science subject; an experimental science; mathematics; one of the creative arts. It is this comprehensive range of subjects that makes the Diploma Programme a demanding course of study designed to prepare students effectively for university entrance. In each of the academic areas students have flexibility in making their choices, which means they can choose subjects that particularly interest them and that they may wish to study further at university.

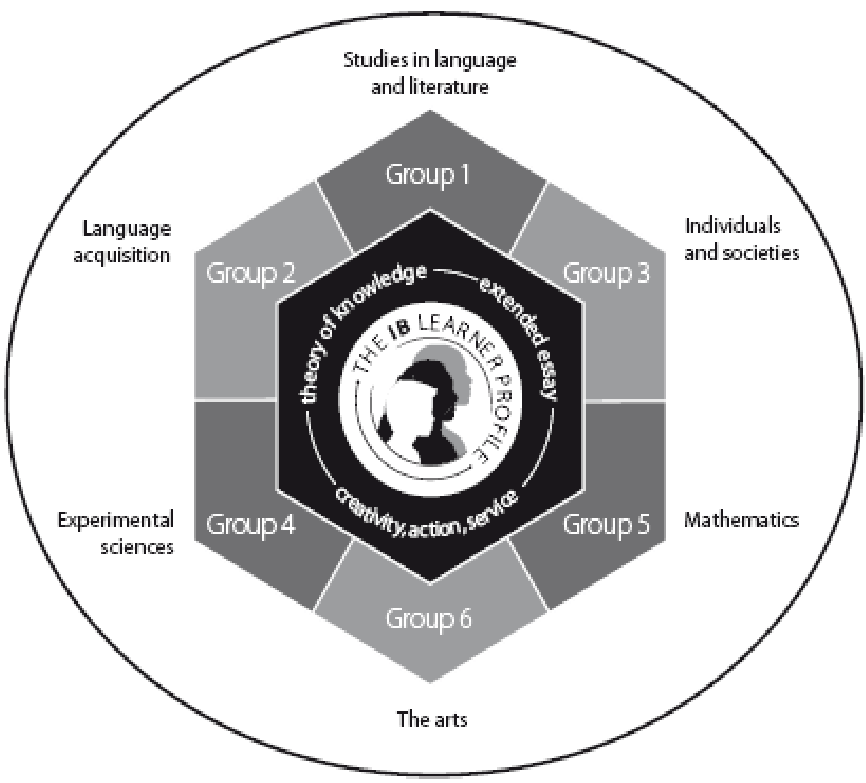

Figure 1
Diploma Programme model

INTERNATIONAL BACCALAUREATE (IB)
MATHEMATICAL STUDIES STANDARD LEVEL, YEARS 1 AND 2
Grades 11, 12

Unit of Credit: 1 Year for Year 1 and 1 Year for Year 2

Pre-requisite: Geometry for Year 1
IB Mathematical Studies Standard Level Year 1 for Year 2

Course Overview:
The Mathematical Studies Standard Level, Years 1 and 2 course syllabus focuses on important mathematical topics that are interconnected. The syllabus is organized and structured with the following tenets in mind: placing more emphasis on student understanding of fundamental concepts than on symbolic manipulation and complex manipulative skills; giving greater emphasis to developing students' mathematical reasoning rather than performing routine operations; solving mathematical problems embedded in a wide range of contexts; using the calculator effectively.

The course includes project work, a feature unique to mathematical studies SL within group 5. Each student completes a project, based on their own research; this is guided and supervised by the teacher. The project provides an opportunity for students to carry out a mathematical study of their choice using their own experience, knowledge and skills acquired during the course. This process allows students to take sole responsibility for a part of their studies in mathematics.

The students most likely to select this course are those whose main interests lie outside the field of mathematics, and for many students this course will be their final experience of being taught formal mathematics. All parts of the syllabus have therefore been carefully selected to ensure that an approach starting from first principles can be used. As a consequence, students can use their own inherent, logical thinking skills and do not need to rely on standard algorithms and remembered formulae. Students likely to need mathematics for the achievement of further qualifications should be advised to consider an alternative mathematics course.

Owing to the nature of mathematical studies SL, teachers may find that traditional methods of teaching are inappropriate and that less formal, shared learning techniques can be more stimulating and rewarding for students. Lessons that use an inquiry-based approach, starting with practical investigations where possible, followed by analysis of results, leading to the understanding of a mathematical principle and its formulation into mathematical language, are often most successful in engaging the interest of students. Furthermore, this type of approach is likely to assist students in their understanding of mathematics by providing a meaningful context and by leading them to understand more fully how to structure their work for the project.

Aims
1. To know and use essential notation, terminology, concepts and principles.
2. To organize, interpret and present information accurately in written, symbolic, tabular, graphical and diagrammatic forms.

3. To present and communicate information processed and analyzed by appropriate mathematical tools.
4. To recognize patterns and structures in a variety of situations and draw inductive generalizations.
5. To demonstrate knowledge of the applications of mathematics to life in a technological society.

Topic 1—Number and Algebra

The aims of this topic are to introduce some basic elements and concepts of mathematics, and to link these to financial and other applications.

- Natural numbers; integers; rational numbers; and real numbers.
- Not required: proof of irrationality, for example, of $\sqrt{2}$.
- Approximation: decimal places, significant figures.
- Percentage errors.
- Estimation.
- Expressing numbers in the form $a \times 10^k$, where $1 \leq a < 10$ and k is an integer.
- Operations with numbers in this form.
- I (Système International) and other basic units of measurement: for example, kilogram (kg), metre (m), second (s), litre (l), metre per second ($m\ s^{-1}$), Celsius scale.
- Currency conversions.
- Use of a GDC to solve
    - pairs of linear equations in two variables
    - quadratic equations.
- Arithmetic sequences and series, and their applications.
- Use of the formulae for the nth term and the sum of the first n terms of the sequence.
- Geometric sequences and series.
- Use of the formulae for the nth term and the sum of the first n terms of the sequence.
- Not required:
- formal proofs of formulae.
- Not required:
- use of logarithms to find n, given the sum of the first n terms; sums to infinity.
- Financial applications of geometric sequences and series:
    - compound interest
    - annual depreciation.
- Not required:
    - use of logarithms.

Topic 2—Descriptive Statistics

The aim of this topic is to develop techniques to describe and interpret sets of data, in preparation for further statistical applications.

- Classification of data as discrete or continuous.
- Simple discrete data: frequency tables.
- Grouped discrete or continuous data: frequency tables; mid-interval values; upper and lower boundaries.
- Frequency histograms.

- Cumulative frequency tables for grouped discrete data and for grouped continuous data; cumulative frequency curves, median and quartiles.
- Box-and whisker diagram - .
- Not required:
    - treatment of outliers.
- Measures of central tendency.
- For simple discrete data: mean; median; mode.
- For grouped discrete and continuous data: estimate of a mean; modal class.
- Measures of dispersion: range, interquartile range, and standard deviation.

<u>Topic 3—Logic, Sets, and Probability</u>

The aims of this topic are to introduce the principles of logic, to use set theory to introduce probability, and to determine the likelihood of random events using a variety of techniques.

- Basic concepts of symbolic logic: definition of a proposition; symbolic notation of propositions.
- Compound statements: implication, $\Rightarrow$; equivalence, $\Leftrightarrow$; negation, $\neg$; conjunction, $\wedge$; disjunction, $\vee$; exclusive disjunction, $\underline{\vee}$.
- Translation between verbal statements and symbolic form.
- Truth tables: concepts of logical contradiction and tautology.
- Converse, inverse, contrapositive.
- Logical equivalence.
- Testing the validity of simple arguments through the use of truth tables.
- Basic concepts of set theory: elements $x \in A$, subsets $A \subset B$; intersection $A \cap B$; union $A \cup B$; complement A' .
- Venn diagrams and simple applications.
- Not required:
    - knowledge of de Morgan's laws.
- Sample space; event A; complementary event, A' .
- Probability of an event.
- Probability of a complementary event.
- Expected value.
- Probability of combined events, mutually exclusive events, independent events.
- Use of tree diagrams, Venn diagrams, sample space diagrams and tables of outcomes.
- Probability using "with replacement" and "without replacement".
- Conditional probability.

<u>Topic 4—Statistical Applications</u>

The aims of this topic are to develop techniques in inferential statistics in order to analyze sets of data, draw conclusions and interpret these.

- The normal distribution.
- The concept of a random variable; of the parameters μ and σ ; of the bell shape; the symmetry about $x = \mu$.
- Diagrammatic representation.

- Normal probability calculations.
- Expected value.
- Inverse normal calculations.
- Not required:
    - Transformation of any normal variable to the standardized normal.
- Bivariate data: the concept of correlation.
- Scatter diagrams; line of best fit, by eye, passing through the mean point.

Year One

Introduction to the Graphical Display Calculator

- Arithmetic calculations
- Graphing functions
- Common buttons
- Data lists

Number and Algebra

- Sets of numbers; natural, integers, rational and real
- Approximation, decimal places, significant figures, percentage error, estimation
- Operations with scientific notation
- Metric system
- Arithmetic sequences and series
- Geometric sequences and series
- Solution of linear equations with GDC
- Solutions of quadratic equations by factorization and GDC

Functions

- Concepts of functions as a mapping, domain, range, mapping diagrams
- Linear functions and their graphs
- Quadratic functions, axis of symmetry, vertex, intercepts

Sets, Logic and Probability

- Basic concepts of set theory; subsets, intersection, union, complement
- Venn diagrams and simple applications
- Sample space and complementary events
- Equally likely events, probability of an event A given by $P(A) = \frac{n(A)}{n(U)}$ probability of a complementary event.
- Venn diagrams, tree diagrams, table of outcomes
- Laws of probability, combined events, mutually exclusive events, independent events, conditional probability

Geometry and Trigonometry

- Coordinates in two dimensions, points, lines, planes, distance formula
- Equations of line in two dimensions, gradient, intercepts parallel and perpendicular lines

- Right-angle trigonometry, trigonometric ratios
- Sine Rules, cosine rule, area of triangle, constructions
- Three dimensional geometry, cubes, prisms, pyramids, cylinders, spheres, cones

Statistics
- Classification of data as discrete or continuous
- Frequency tables and polygons
- Histograms stem and leaf diagrams, boundaries
- Cumulative frequency tables and graphs, box and whisker plots, percentiles, quartiles
- Measures of central tendency, mean, median, mode
- Measures of dispersion, range, interquartile range, standard deviation

Financial Mathematics
- Currency conversions
- Simple interest
- Compound interest, depreciation
- Construction and use of tables, loan and repayment schemes, investment and saving schemes, inflation

Year Two

Sets, Logic and Probability (continued)
- Concepts of symbolic logic, definition of proposition, notation
- Truth tables
- Definition of implication, converse, inverse and contrapositive
- Functions (continued)
- Exponential functions, growth and decay, asymptotic behavior
- Sine and cosine functions, amplitude, period
- Accurate graph drawing
- Use of GDC to sketch and analyze new functions. Statistics (continued)
- Scatter plots, line of best fit, bivariate data, Pearson's product-moment correlation coefficient, interpretation of correlations
- The regression line for y on x, use of regression line for predictions
- The Chi-Square test for independence, formulation of null and alternative hypothesis, significance levels, contingency tables, expected frequencies, degrees of freedom

Differential Calculus
- Gradient of two points on the graph of a function, behavior of the gradient as one point approaches other, tangent to a curve
- Derivatives of 1-variable monomials and polynomials
- Gradients of curves for given values of x, values of x where f '(x) is given, equations of the tangent at a given point
- Increasing and decreasing functions, graphical interpretation of derivatives
- Values of x where the gradient is zero, local maxima and minima

Project

- This is a significant piece of written work for which the student undertakes personal research on a mathematical project of their choice. This project, which is undertaken in the first semester of the second year, contributes the internal part of their IB math assessment.

Syllabus Review and Exam Preparation

- At the conclusion of the course in the final semester of the second year, students undertake a comprehensive review of the course material alongside preparation and practice for examinations.

INTERNATIONAL BACCALAUREATE (IB)
MATH STANDARD LEVEL, YEARS 1 AND 2
Grades 11, 12

Unit of Credit: 1 Year for Year 1 and 1 Year for Year 2

Pre-requisite: Algebra 2 for Year 1
IB Math Standard Level Year 1 for Year 2

Course Overview:
This course is aimed at students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

The course focuses on introducing important mathematical concepts through the development of mathematical techniques. The intention is to introduce students to these concepts in a comprehensible and coherent way, rather than insisting on mathematical rigor. Students should, wherever possible, apply the mathematical knowledge they have acquired to solve realistic problems set in an appropriate context.

The internally assessed component of this course, the portfolio, offers students a framework for developing independence in their mathematical learning by engaging in mathematical investigation and mathematical modeling. Students are provided with opportunities to take a considered approach to these activities and to explore different ways of approaching a problem. The portfolio also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

This course does not have the depth found in the IB Mathematics Higher Level course. Students wishing to study subjects with a high degree of mathematical content should therefore opt for the Mathematics Higher Level course rather than a Mathematics Standard Level course.

Aims
1. To appreciate the multicultural and historical perspectives of all group five courses.
2. To enjoy the courses and develop an appreciation of the elegance, power, and usefulness of the subjects.
3. To develop logical, critical and creative thinking.
4. To develop an understanding of the principles and nature of the subject.
5. To employ and refine their powers of abstraction and generalization.
6. To develop patience and persistence in problem solving.
7. To appreciate the consequences arising from technological developments.
8. To transfer skills to alternative situations and to future developments.
9. To communicate clearly and confidently in a variety of contexts.

Topic 1—Number and Algebra

The aim of this section is to introduce students to some basic algebraic concepts and applications.

- Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.
- Sigma notation.
- Applications
- Elementary treatment of exponents and logarithms.
- Laws of exponents; laws of logarithms.
- Change of base.
- The binomial theorem: expansion of $(a + b)n$, $n \in$N.
- Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$
- **Not required:** formal treatment of permutations and formula for ${}_{n}P_{r}$

<u>Topic 2—Functions and equations</u>

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$

- Concept of function $f : x \mapsto f(x)$
- Domain, range; image (value).
- Composite functions.
- **Not required:** Domain restriction.
- Identity function. Inverse function f^{-1} .
- The graph of a function; its equation $y = f(x)$.
- Function graphing skills.
- Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.
- Use of technology to graph a variety of functions, including ones not specifically mentioned.
- The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$
- Transformations of graphs.
- Translations: $y = f(x) + b$; $y = f(x - a)$.
- Reflections (in both axes): $y = -f(x)$; $y = f(-x)$
- Vertical stretch with scale factor p: $y = pf(x)$
- Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$.
- Composite transformations. The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry.
- The form $x \mapsto a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q, 0)$.
- The form $x \mapsto a(x - h)^2 + k$, vertex (h, k) .
- The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.

- The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.
- Vertical and horizontal asymptotes.
- Exponential functions and their graphs: $x \mapsto a^x, a > 0, x \mapsto e^x$
- Logarithmic functions and their graphs: $x \mapsto log_a x,\ x > 0, x \mapsto \ln x, x > 0$.
- Relationships between these functions:
- $a^x = e^{xlna}$; $log_a a^x = x, a^{log_a x} = x, x > 0$.
- Solving equations, both graphically and analytically.
- Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.
- Solving $ax^2 + bx + c = 0, a \neq 0$. The quadratic formula.
- The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
- Solving exponential equations.
- Applications of graphing skills and solving equations that relate to real-life situations.

Topic 3-Circular functions and trigonometry

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

- The circle: radian measure of angles; length of an arc; area of a sector.
- Definition of $cos\theta$ and $sin\theta$ in terms of the unit circle.
- Definition of $tan\theta$ as $\frac{sin\theta}{cos\theta}$.
- Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.
- The Pythagorean identity $cos^2\theta + sin^2\theta = 1$.
- Double angle identities for sine and cosine.
- Relationship between trigonometric ratios.
- The circular functions $sin\ x$, $cos\ x$ and $tan\ x$: their domains and ranges; amplitude, their periodic nature; and their graphs.
- Composite functions of the form $f(x) = asin(b(x + c)) + d$.
- Transformations
- Applications
- Solving trigonometric equations in a finite interval, both graphically and analytically.
- Equations leading to quadratic equations in sin x, cos x or tan x .
- **Not required:** the general solution of trigonometric equations.
- Solution of triangles.
- The cosine rule.
- The sine rule, including the ambiguous case.
- Area of a triangle, $\frac{1}{2}absinC$.
- Applications.

Topic 4—Vectors

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

- Vectors as displacements in the plane and in three dimensions
- Components of a vector; column representation $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1 i + v_2 j + v_3 k$.
- Algebraic and geometric approaches to the following:
    - the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;
    - multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;
    - magnitude of a vector, $\boldsymbol{v}$;
    - unit vectors; base vectors; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$;
    - position vectors $\overrightarrow{OA} = a$;
    - $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$.
- The scalar product of two vectors.
- Perpendicular vectors; parallel vectors.
- The angle between two vectors.
- Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.
- The angle between two lines.
- Distinguishing between coincident and parallel lines.
- Finding the point of intersection of two lines.
- Determining whether two lines intersect.
- Expected value.
- Inverse normal calculations.
- Not required:
    - Transformation of any normal variable to the standardized normal.
- Bivariate data: the concept of correlation.
- Scatter diagrams; line of best fit, by eye, passing through the mean point.

Topic 5—Statistics and Probability

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms "write down", "find" and "calculate" will be used.

- Concepts of population, sample, random sample, discrete and continuous data.
- Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers.
- Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class.
- **Not required:** frequency density histograms.
- Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles.
- Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data.

- Applications.
- Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.
- Linear correlation of bivariate data.
- Pearson's product–moment correlation coefficient *r*.
- Scatter diagrams; lines of best fit.
- Equation of the regression line of *y* on *x*. Use of the equation for prediction purposes. Mathematical and contextual interpretation.
- **Not required:** the coefficient of determination *R*2.
- Concepts of trial, outcome, equally likely outcomes, sample space (*U*) and event.
- The probability of an event *A* is $P(A) = \frac{n(A)}{n(U)}$.
- The complementary events *A* and *A*'(not *A*).
- Use of Venn diagrams, tree diagrams and tables of outcomes.
- Combined events, $P(A \cup B)$.
- Mutually exclusive events: $P(A \cap B) = 0$.
- Conditional probability; the definition
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
- Independent events; the definition P(*A*| *B*) = P(*A*) = P(*A*| *B*') .
- Probabilities with and without replacement.
- Concept of discrete random variables and their probability distributions.
- Expected value (mean), E(*X*) for discrete data.
- Applications.
- Binomial distribution.
- Mean and variance of the binomial distribution.
- **Not required:** formal proof of mean and variance.
- Normal distributions and curves.
- Standardization of normal variables (*z*-values, *z*-scores).
- Properties of the normal distribution.

<u>Topic 5—Calculus</u>

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications

- Informal ideas of limit and convergence.
- Limit notation.
- Definition of derivative from first principles as $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- Derivative interpreted as gradient function and as rate of change.
- Tangents and normals, and their equations.
- **Not required:** analytic methods of calculating limits.
- Derivative of $x^n (n \in \mathbb{Q}), \sin x, \cos x, \tan x, e^x$, $\ln x$
- Differentiation of a sum and a real multiple of these functions.
- The chain rule for composite functions. The product and quotient rules.
- The second derivative.
- Extension to higher derivatives.

Year One

Introduction to the Graphical Display Calculator

- Arithmetic calculations
- Graphing functions
- Common buttons
- Data lists

Number and Algebra

- Sets of numbers; natural, integers, rational and real
- Approximation, decimal places, significant figures, percentage error, estimation
- Operations with scientific notation
- Metric system
- Arithmetic sequences and series
- Geometric sequences and series
- Solution of linear equations with GDC
- Solutions of quadratic equations by factorization and GDC

Functions

- Concepts of functions as a mapping, domain, range, mapping diagrams
- Linear functions and their graphs
- Quadratic functions, axis of symmetry, vertex, intercepts

Sets, Logic and Probability

- Basic concepts of set theory; subsets, intersection, union, complement
- Venn diagrams and simple applications
- Sample space and complementary events
- Equally likely events, probability of an event A given by $P(A) = \frac{n(A)}{n(U)}$ probability of a complementary event.
- Venn diagrams, tree diagrams, table of outcomes
- Laws of probability, combined events, mutually exclusive events, independent events, conditional probability

Geometry and Trigonometry

- Coordinates in two dimensions, points, lines, planes, distance formula
- Equations of line in two dimensions, gradient, intercepts parallel and perpendicular lines
- Right-angle trigonometry, trigonometric ratios
- Sine Rules, cosine rule, area of triangle, constructions
- Three dimensional geometry, cubes, prisms, pyramids, cylinders, spheres, cones

Statistics

- Classification of data as discrete or continuous
- Frequency tables and polygons
- Histograms stem and leaf diagrams, boundaries
- Cumulative frequency tables and graphs, box and whisker plots, percentiles, quartiles

- Measures of central tendency, mean, median, mode
- Measures of dispersion, range, interquartile range, standard deviation

Financial Mathematics

- Currency conversions
- Simple interest
- Compound interest, depreciation
- Construction and use of tables, loan and repayment schemes, investment and saving schemes, inflation

Year Two

Sets, Logic and Probability (continued)

- Concepts of symbolic logic, definition of proposition, notation
- Truth tables
- Definition of implication, converse, inverse and contrapositive
- Functions (continued)
- Exponential functions, growth and decay, asymptotic behavior
- Sine and cosine functions, amplitude, period
- Accurate graph drawing
- Use of GDC to sketch and analyze new functions. Statistics (continued)
- Scatter plots, line of best fit, bivariate data, Pearson's product-moment correlation coefficient, interpretation of correlations
- The regression line for y on x, use of regression line for predictions
- The Chi-Square test for independence, formulation of null and alternative hypothesis, significance levels, contingency tables, expected frequencies, degrees of freedom

Differential Calculus

- Gradient of two points on the graph of a function, behavior of the gradient as one point approaches other, tangent to a curve
- Derivatives of 1-variable monomials and polynomials
- Gradients of curves for given values of x, values of x where $f'(x)$ is given, equations of the tangent at a given point
- Increasing and decreasing functions, graphical interpretation of derivatives
- Values of x where the gradient is zero, local maxima and minima

Project

- This is a significant piece of written work for which the student undertakes personal research on a mathematical project of their choice. This project, which is undertaken in the first semester of the second year, contributes the internal part of their IB math assessment.

Syllabus Review and Exam Preparation

- At the conclusion of the course in the final semester of the second year, students undertake a comprehensive review of the course material alongside preparation and practice for examinations.

< APPENDICES >

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APPENDIX I

Mathematics Terminology

Assessment: the process of gathering information on the quality of a product, performance, or demonstration.

Clusters: groups of related standards within a domain. Because mathematics is a connected subject, standards from different clusters may sometimes be closely related.

Curriculum: the Montana Common Core Standards in this document that are taught in the mathematics classroom.

Differentiated Instruction: a variety of instructional approaches that address the needs and learning styles of individual students in one-on-one, small group, and large group settings.

Domains: large groups of related standards. Standards from different domains may sometimes be closely related.

Learning Targets: clear and usable statements of intended learning, which are taught to students and which students are expected to learn at specific grade levels or in specific classes, leading to the mastery of benchmarks and standards.

Materials: the resources used to support the teaching of the curriculum. Examples include textbooks, websites, manipulatives, etc.

Mathematical Practice Standards: varieties of expertise that mathematics educators at all levels should seek to develop in their students. Examples include problem solving, reasoning and proof, representation and communication, connections, conceptual understanding, and fluency.

Standards: for each domain and cluster are specific statements that specify what students should understand and be able to do at the end of a particular grade level or by the time they graduate from high school.

APPENDIX II

Common Core State Standards

Understanding Mathematics

The Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Key Points in Mathematics

- The K-5 standards provide students with a *solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions, and decimals*—which help young students build the foundation to successfully apply more demanding math concepts and procedures, and move into applications.

- In kindergarten, the standards follow successful international models and recommendations from the National Research Council's Early Math Panel report, by focusing kindergarten work on the number core: learning how numbers correspond to quantities, and learning how to put numbers together and take them apart (the beginnings of addition and subtraction).

- The K-5 standards build on the best state standards to provide detailed guidance to teachers on how to navigate their way through knotty topics such as *fractions, negative numbers, and geometry*, and do so by maintaining a continuous progression from grade to grade.

- The standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels - rather than the current practices by which many students learn enough to get by on the next test, but forget it shortly thereafter, only to review again the following year.

- Having built a strong foundation K-5, students can do hands on learning in geometry, algebra, and probability and statistics. Students who have completed 7^{th} grade and mastered the content and skills through the 7^{th} grade will be *well-prepared for algebra* in 8^{th} grade.

- The middle school standards are robust and provide a coherent and rich *preparation for high school mathematics*.

- The high school standards call on students to *practice applying mathematical ways of thinking to real world issues and challenges*; they prepare students to think and reason mathematically.

- The high school standards set a *rigorous definition of college and career readiness*, by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.

- The high school standards *emphasize mathematical modeling*, the use of mathematics and statistics to analyze empirical situations, understand them better, and improve decisions. For example, the draft standards state: “Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. It is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.”

Modeling Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

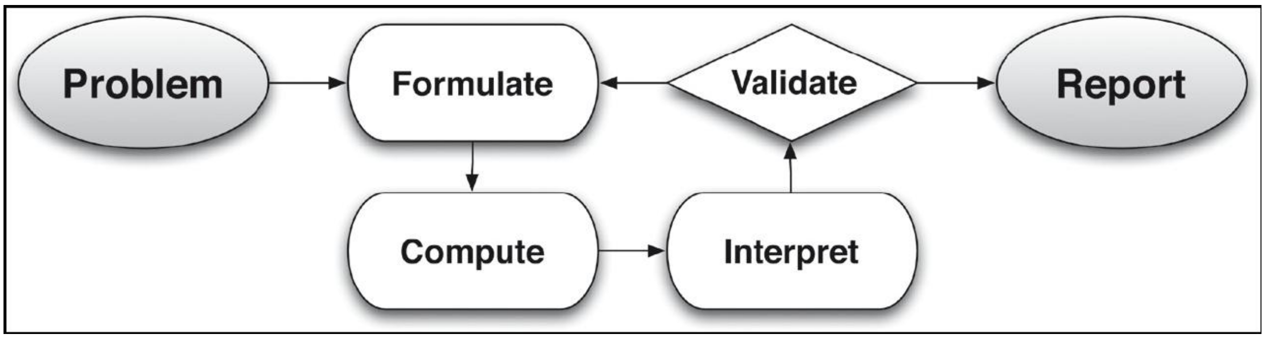
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed;
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player;
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car;
- Modeling savings account balance, bacterial colony growth, or investment growth;
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport;
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism;
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).*

APPENDIX III

Common Core Standards for Mathematics Glossary
http://www.corestandards.org/Math

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	**3 x ? = 18, and 18 ÷ 3 = ?**	**? x 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The properties of operations. Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $^1/a$ so that $a \times {}^1/a = {}^1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

TABLE 4. The properties of equality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 5. The properties of inequality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this glossary.

Associative property of multiplication. See Table 3 in this glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Identity property of 0. See Table 3 in this Glossary.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

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[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

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[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

APPENDIX IV

Research-based Interventions

Interventions provide direct, explicit instruction focused on increasing skill levels by actively involving the student. Interventions include methods such as providing additional instruction, change of instruction (i.e., teaching a different or additional math program), or a specific behavior modification program. Effective interventions increase the success of instruction by:

- Increasing instructional time
- Decreasing number of students in an instructional group
- Improving the quality of instruction
- Increasing the frequency of instruction by giving more opportunities for reteaching, review, and supervised practice
- Focusing on the most essential learning needs (the underlying skill deficit).
- Providing explicit and systematic instruction.
- Using additional methodology

APPENDIX V

Checklist for Effective Instruction

Grouping for Instruction

Group students based on instructional purpose (e.g., one-on-one, pairs, small group) and students' needs (e.g., use small same-ability groups for struggling readers)

- ☐ *Use flexible grouping to allow students to be members of more than one group*

Explicit and Systematic Instruction

Review previous learning and prerequisite knowledge and skills

- ☐ *Keep reviews brief, frequent, and spaced over time*

Re-teach when necessary

- ☐ *Use multiple techniques and vary presentation/format from initial instruction*

Identify objective and specific elements to be learned

- ☐ *Identify and communicate learning targets*
- ☐ *Build specific knowledge and skills identified in state standards*
- ☐ *Target needs based on continuous progress monitoring*

Activate and build background knowledge

- ☐ *Build on what students already know and expand their knowledge base*
- ☐ *Consider cultural and linguistic diversity*

Reduce the amount of new information presented at one time

- ☐ *Use a logical sequence (e.g., progress from easier to more complex)*

Model or demonstrate procedures

- ☐ *Show how something is done*
- ☐ *Think aloud and explain thinking processes used*
- ☐ *Provide opportunities for students to communicate their reasoning and thinking (ex: math talk)*

Provide examples and, when appropriate, non-examples

- ☐ *Include visual prompts and/or graphic organizers*

Maximize students' engagement

- ☐ *Include a variety of ways for students to participate (e.g., response cards)*
- ☐ *Pace instruction, stop to repeat key ideas, and allow extra time if needed*

Check for students' understanding

- ☐ *Ask different levels of questions and encourage students to generate questions*
- ☐ *Incorporate sufficient wait time*
- ☐ *Provide corrective feedback to help students understand*
- ☐ *Adjust instruction so students are challenged and able to develop new skills*

Scaffolding Practice

Provide opportunities to practice with teacher support and guidance

- ☐ *Use appropriate level of materials*
- ☐ *Incorporate manipulatives, graphic organizers, and/or hands-on activities*
- ☐ *Provide appropriate support for the level of proficiency*

Check for understanding

- ☐ *Provide prompts to help students notice, find, and correct errors*
- ☐ *Help students learn to self-monitor for understanding*
- ☐ *Clarify misconceptions; re-teach when necessary*
- ☐ *Provide positive, motivating feedback*

Provide many opportunities for independent practice to promote automaticity, generalization to different contexts, and maintenance

- ☐ *Initially provide support during independent practice*
- ☐ *Integrate practice of new knowledge/skills with those previously taught*
- ☐ *Make connections across the curriculum*
- ☐ *Frequently monitor students working independently to prevent them from practicing errors*

Progress Monitoring

Regularly use a classroom-based instructional assessment or progress-monitoring system to inform instruction

Determine if students are making expected progress and if instruction needs adjusting (e.g. regrouping more intense instruction

APPENDIX VI

Concrete-Representational-Abstract Strategy

Webster's says "a poem is concrete; poetry is abstract." Though we have understood the value and used Piaget's concrete stage for decades, we often do not recognize that a large part of the value in making an approach concrete is to assist students in moving to higher level thinking.

The Concrete-Representational-Abstract Strategy (C-R-A Strategy) provides an organizational structure within which lessons can be designed to effectively help students reach an abstract level of thinking around difficult concepts and content. The Concrete-Representational-Abstract Strategy focuses on helping students move in sequence through the following phases.

Concrete: When introducing new concepts to students, describe and model the concept using concrete materials. The concrete level of understanding is the most basic level of cognitive skill. When students have mastered this level of concrete understanding by modeling and describing the concept concretely themselves, instruction moves to the next step. In math, this could involve using manipulatives to count, measure, or sort.

Representational: When students have mastered understanding on the concrete level, describe and model the concept by drawing or using other pictures that represent the concrete objects. The ability to represent concepts is the next level of cognitive skill. When students are able to demonstrate the concept representationally themselves, instruction moves to the next step. In math, this could involve using tally marks or dots to count, measure, or sort.

Abstract: When students have mastered the representational level, describe and model the concept in an abstract manner, using the typical form in which problems are presented. The ability to work abstractly is the most advanced cognitive level. In math, this could involve using numbers to count, measure, or sort.

Note: C-R-A is dependent on the number of times a student successfully demonstrates a skill at each cognitive level before moving to a higher level. This strategy is especially effective for mathematics, science, and social studies, where students learn abstract concepts that are readily adaptable to concrete materials. It is important that each step is modeled and described by the instructor, and that students get immediate feedback at each step to correct any misconceptions along the way.

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APPENDIX VII

Assessment

Assessment is an important tool in any mathematics program. Assessment has traditionally been thought of as a test to measure students' attainment of specified goals. Although this outcome is one purpose of assessment, assessment also serves other purposes, such as guiding teachers' instructional practice and assessing FOR learning. Research supports the precept that students' learning is improved when assessment is an integral part of ongoing classroom activities (Black and William 1998). Good Formative and Summative assessments provide useful information about students' learning, as well as for instructional practice.

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Formative assessment, done well, represents one of the most powerful instructional tools available to a teacher or a school for promoting student achievement. Teachers and schools
can use formative assessment to identify student understanding, clarify what comes next in their learning, trigger and become part of an effective system of intervention for struggling students, inform and improve the instructional practice of individual teachers or teams, help students track their own progress toward attainment of standards, motivate students by building confidence in themselves as learners, fuel continuous improvement processes across faculties, and, thus, drive a school's transformation (Stiggins and DuFour, 2009). Maximizing the Power of Formative Assessments

Assessments before and during instruction allow teachers to make appropriate decisions about such considerations as reviewing materials, reteaching a difficult concept, or providing additional or more challenging material for students who are struggling or need enrichment. Teachers use many assessment techniques, including:

- open ended questions
- constructive response tasks
- selected response items
- performance tasks
- teacher observations
- class discussions
- journals
- portfolios

Teachers use district benchmark assessments and summative assessments to evaluate student progress toward meeting grade level standards.

In addition, MCPS administers the Common Core Assessment (the CRT through 2014 and the Smarter-Balanced Assessment beginning in 2015) to all students in grades 3-8 and 10 (CRT) or 11 (Smarter-Balanced Assessment) to monitor overall student progress in mathematics.

In order to meet the needs and interests of the highly motivated and accelerated student, a number of Advanced Placement mathematics courses are offered. College credit may be earned upon the successful completion of the subject specific AP examination, in accordance with the registration practices of the individual post-secondary institutions. A student can become eligible for enrollment in these courses through teacher nomination, counselor recommendation, or parent request.

APPENDIX VIII

Gifted Education

The term "Gifted and Talented children" means children of outstanding abilities who are capable of high performance and require differentiated educational programs beyond those normally offered in public schools in order to fully achieve their potential contribution to self and society. Therefore, it is essential that the most appropriate delivery of curriculum and instruction be used when serving the needs of *all students* and must be differentiated or appropriately accommodated for high-ability students.

The MCPS Common Core Math curriculum provides structure for teachers which allow for differentiated instruction for students who are not proficient as well as those who are gifted and/or high-achieving math learners. Differentiating the curriculum will allow high-ability students to achieve their potential in mathematics.

Classroom management and effective instructional practices are essential in serving all students and require differentiation for high-ability students as well. This refers to how children will be grouped to receive instruction in content knowledge, skills and processes.

Appropriate grouping for gifted and talented students includes **individualization** (modifying the instruction, materials, and learning targets for an individual student to allow them to meet their potential), **cluster grouping by ability or achievement** (to provide differentiated instruction in heterogeneous groups), and **acceleration** (single subject acceleration and AP classes).

Students in prekindergarten through 5th grade should be served with the cluster grouping model and differentiated instruction. For some students, whole grade or single subject acceleration may be appropriate.

Middle school students can also be served with the cluster grouping model as well as through individualization whereby a student can move through the curriculum at a pace more suited to his or her ability. At the middle school level an Accelerated Track, compacts grades 7, 8, and high school Algebra I into two years, moving at a more efficient pace for high-ability learners.

High school students have the opportunity to enroll in a variety of course options that mirror college level rigor. These include Advanced Placement courses, dual enrollment offerings, International Baccalaureate courses, and the University of Montana's Jump Start program.

APPENDIX IX

Library Media

The library media specialist has an essential role in teaching and learning. As information specialist, the library media specialist working collaboratively with teachers, administrators, and parents:

- Provides knowledge of availability and suitability of information resources to support curriculum initiatives. This is particularly relevant with the Indian Education for All Law.
- Engages in the developmental process with the planning team, using knowledge of school curriculum resources.
- Serves as an expert in organizing, synthesizing, and communicating information. Acquisition, organization, and dissemination of resources to support the curricular areas through the library media center are cost-effective for the entire school district.
- Teachers and library media specialists share responsibility for reading and information literacy instruction. They plan and teach collaboratively based on the needs of the student. (adapted from ALA statements, 1999)

APPENDIX X

Family and Community Involvement

Forming strong relationships with families and the community is important to a successful mathematics program. Research supports the conclusion that adults' attitudes toward our children's education, and their involvement in it, have a significant impact on classroom success.

The mathematics classroom today may look very different from the classrooms that adults experienced when they were in school. Some adults may even feel uncomfortable or have misconceptions about the mathematics our children are learning. Therefore, it is important for MCPS to provide parents and community members with information that will help them understand the mathematics program and ways they can contribute to the success of our children's learning.

MCPS will:

1. Inform parents and the community of the goals of the mathematics programs, why these goals are important, and what our children will be learning.
2. Provide information on how parents can support their children's learning in school and at home.
3. Offer hands-on experiences for parents so that they understand and appreciate the mathematics their children are learning.
4. Communicate to parents and the community the expectation that all students can be successful in mathematics.

As adults, we can help our children develop a good attitude about mathematics. We are sometimes heard to say, "I can't do math" or "I don't like math." By contrast, we are much less likely to say, "I can't read." Adults should understand that their feelings about mathematics can affect our children's thinking about math and about themselves as mathematicians.

Mathematical literacy is just as important as reading literacy.

By understanding the goals of the MCPS mathematics program and the reasons that these goals are important, parents and community members can also serve as advocates for our schools. Collaborating with parents and the community and inviting them to participate in efforts to improve the mathematics program are essential to increasing achievement for all MCPS students.

APPENDIX XI

Comprehension Strategies

Proficient readers use these stategies before, during and after reading:

Activate Background Knowledge

- What do you already know about this topic?
- What connections(schema) can you make to your life, the world or other things you have read?

Ask Questions

- What do you want to know about this topic?
- What questions come up as you read?

Infer

- What background knowledge and explicit information from the text are you using to make meaning?
- What questions come up as you read?

Determine Importance

- What words, sentences, ideas, and themes are especially important?
- What is the big picture, the main idea?

Make Mental Images

- What images come to mind as you read?

Synthesize

- What inferences and key concepts are you putting together to deepen your understanding?

Monitor Comprehension

- Where does your comprehension break down?
- What causes the difficulty?
- How can you fix it?

√Reread √Read ahead √Use Context Clues
√Restate √Research √Check Pictures & Graphics
√Use Decoding Strategies (Sound it out)

MCPS

Missoula County Public Schools

APPENDIX XII

Teaching about Controversial Issues

Missoula County Public Schools

BOARD POLICY - INSTRUCTION **#2330**

Academic Freedom

The Board recognizes and supports Academic Freedom as necessary for an environment conducive to the free exchange of ideas and learning.
Academic Freedom is the view that if teachers are to promote the growth of knowledge, they require the freedom to teach and conduct inquiry without fear of sanction or reprisals should they present an unpopular or controversial idea.
Teachers shall help students learn to objectively and respectfully examine differences of opinion, analyze and evaluate facts and their sources, and form their own reasoned judgments about the relative value of competing perspectives.

The Board directs the teaching staff to:

• Refrain from using one's classroom position to promote one's own ideology or any partisan point of view.
• Ensure that issues presented pertain to course objectives.
• Provide students opportunities to develop critical thinking: that is the ability to detect propaganda and to distinguish between fact, opinion and misinformation.
• Respect each student's right to form, choose, hold and/or change an opinion or belief.
• Create an environment in which students are free to form judgments independently.

Any person may file complaints pursuant to this policy through Board Policy 4310P, the uniform grievance procedure.

This policy may not be used to challenge educational materials themselves. Please see:

BP 2313 Dealing with Challenged Educational Resources
BP 2313P Procedure for Dealing with Challenged Educational Resources

Legal Reference:	Article X, Sec. 8, Montana Constitution - School district trustees	
	§ 20-3-324(16) and (17), MCA	Powers and duties

Policy History:

Adopted on: January 14, 2003
Revision presented to PN&P Committee on March 25, 2009
Approved on first reading: May 12, 2009
Posted for public comment until: July 22, 2009
Adopted on second reading: August 11, 2009

APPENDIX XIII

Meeting Students' Diverse Needs

Students with diverse needs--those with unique abilities and/or disabilities--will have differentiated opportunities to achieve competencies and standards, and at rates and in manners consistent with their needs. Students who excel will have opportunities to achieve competencies and standards at a faster pace. Some appropriate modifications include the following:

Content Enrichment is the presentation of curricula in more depth and breadth. This may include extra lessons or assignments used to elaborate the student's richness of understanding of existing curriculum competencies and/or standards.

Content Sophistication is the presentation of curricula that most students might not be able to master.

Content Novelty is the presentation of content not covered in traditional school curriculum.

Content Acceleration is the presentation of curricula intended for older students and/or those in higher grades. This may include accelerating a student through the entire grade level curriculum and into the curriculum of the next grade level.

The needs of those students who have difficulty learning concepts will be met in a variety of ways in the classroom both through informal intervention and formally prescribed intervention as necessary. Among possible accommodations are the following:

Supplementary materials such as study guides or materials available at easier reading levels covering the same content could be used. Books on tape are also available for some subjects.

Class notes could be provided to students with special needs. Notes specific to tests are particularly helpful.

A **variety of instructional approaches** should be used to meet needs of visual, auditory, and kinesthetic learners.

The **amount of material tested at one time** could be reduced.

Assistance from Resource Room or Title I Staff should be employed as necessary.

Alternative and/or modified assignments should be employed as necessary. For example, the assignment of projects rather than reports or the opportunity for some students to dictate answers to questions.

Questions could be read aloud for those students who are more effective at auditory learning.

Taping class lectures could be used to help those students who have difficulty writing or comprehending.

APPENDIX XIV

References

Achieve the Core Mathematics website, http://www.achievethecore.org/math-common-core/math-shifts/

Berkas, Nancy, Cynthia Pattison. "Closing the Achievement Gap The Passion to Teach Significant Mathematics." NCTM News Bulletin (2006): 4.

Buchholz, Lisa. "Learning Strategies for Addition and Subtraction Facts: The Road to Fluency and the License to Think." Teaching Children Mathematics March(2004): 362-367.

Cai, Jinfa. "Developing Algebraic Reasoning in the Elementary Grades." Teaching Children Mathematics 5(1998): 225-229.

Charles, Randall, Joanne Lobato. Future Basics: Developing Numerical Power. Golden, CO: NCSM, 2000.

Classzone website, www.classzone.com

Common Core State Standards Initiative Mathematics website, http://www.corestandards.org/Math

Daniels, Harvey, et al. Best Practice. 3rd. Portsmouth, NH: Heinemann, 2005.

Edwards, Thomas G.. "Some 'Big Ideas' of Algebra in the Middle Grades." Mathematics Teaching in the Middle School 6(2000): 26-31.

Figure This! Website, http://www.figurethis.org/index.html

Khan Academy Website, https://www.khanacademy.org/

Losq, Christine. "Number Concepts and Special Needs Students: The Power of Ten-Frame Tiles." Teaching Children Mathematics February(2005): 310-315.

Lott, Johnny W.. "Correcting the Course of math Education." PL January(2007): 28-31.

Math.com Website, http://www.math.com/

McCrone, Sharon Soucy. "Mathematical Literacy-It's Become Fundamental." PL January(2007): 32-37.

Mirra, Amy. Administrator's Guide How to Support and Improve Mathematics Education in Your School. Reston, Virginia: NCTM, 2003.

Montana Office of Public Instruction, "Montana Standards for Mathematics." www.opi.mt.gov. October 1998. MT OPI. 1Apr 2007 <www.opi.mt.gov>.

Montana Office of Public Instruction, "Montana Common Core Standards Mathematical Practice and Content." www.opi.mt.gov November, 2011. MT.

NCTM Illuminations Website, http://illuminations.nctm.org/

NCTM, "Navigating through Algebra." Navigation Series Grades 6-8 (2000): 1-8.

NCTM, "Navigating through Data Analysis." Navigation Series Grades 6-8 (2000): 1-8.

NCTM, "Navigating through Geometry." Navigation Series Grades 6-8 (2000): 1-8.

NCTM, "Navigating through Measurement." Navigation Series Grades 6-8 (2000): 1-9.

Northwest Regional Educational Lab, Improving Adolescent Mathematics: Findings From Research. Portland, OR: NREL,

Reys, Robert E.. "Computation versus Number Sense." Mathematics Teaching in the Middle School 4(1998): 110-112.

Rothstein, Andrew, Evelyn Rothstein. "Writing and Mathematics: An Exponential Combination." PL January(2007): 21-25.

Schielack, Jane, et al. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics. Reston, Virginia: NCTM, 2006.

Seymore, Dale, Developing Mathematical Ideas Resources, http://toolkitforchange.org/toolkit/view.php?obj=1041&link=653

Smarter Balanced Assessment Consortium Website, Sample Items and Performance Tasks http://www.smarterbalanced.org/sample-items-and-performance-tasks/

Steen, Lynn Arthur. "Every Teacher Is a Teacher of Mathematics." PL January(2007): 16-20.

Stiggins, Richard, et al. Classroom Assessment for Student Learning. Portland, Oregon: Assessment Training Institute, Inc., 2004.

Stigler, James, James Hiebert. "Improving Mathematics Teaching." Educational Leadership February (2004): 12-17.

Strong, Richard, et al. "Creating a Differentiated Mathematics Classroom." Association for Supervision and Curriculum Development (2004): 73-77.

That Quiz Website, www.thatquiz.org

Tilly, III, W. David. "Response to Intervention: An Overview What Is It? Why Do It? Is It Worth It?." The Special Edge Winter/Spring(2006): 1-10.

APPENDIX XV

Adopted Materials

Grades Kindergarten and 1-2
Math Expressions, Houghton Mifflin, 2013

Grades 3-5
Math Expressions, Houghton Mifflin, 2009

Middle School
Grade 6, Math Course 1, McDougal-Littell, 2007
Grade 7, Math Course 2, McDougal-Littell, 2007
Grade 8, Math Course 3, McDougal-Littell, 2007

Algebra Readiness, McDougal-Littell, 2008

Algebra 1, McDougal-Littell, 2008
Geometry, McDougal-Littell, 2008

High School
Algebra 1, Algebra 1, McDougal-Littell, 2008
Geometry, Geometry, McDougal-Littell, 2008
Algebra 2, Algebra 2, McDougal-Littell, 2008

Functions, Statistics, & Trigonometry, UCSMP Functions, Statistics, and Trigonometry, McGraw-Hill, 2010
Technical Math MAT111, Mathematics for the Trades, 9th Ed, Pearson, 2011
Discrete Math MAT115, UCSMP Functions, Statistics, and Trigonometry, McGraw-Hill, 2010
Pre-Calculus, Precalculus: Mathematics for Calculus, 5th Ed, by Stewart, Thomson
Advanced Placement Statistics, The Practice of Statistics, 2008
The Platinum Notebook, by Yates, Freeman Publishing
Advanced Placement Calculus AB, Single Variable Calculus: Early Transcendentals, 6th Ed., by Stewart, Thomson publishers
Advanced Placement Calculus BC, Single Variable Calculus: Early Transcendentals, 6th Ed., by Stewart, Thomson Publishers
IB Mathematical Studies Standard Level, Pearson Baccalaureate Standard Level Mathematics, Prentice Hall, 2008
IB Math Standard Level, Mathematical Studies, Pearson
Single Variable Calculus Early Transcendentals, 6th Ed., Brooks & Cole, 2008
Mathematics for the International Student Mathematical Studies SL, 1st Ed., Haese & Harris Publications, 2008

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